An adjoint-weighted principal components approach for determining dominant atmospheric drivers of ocean variability

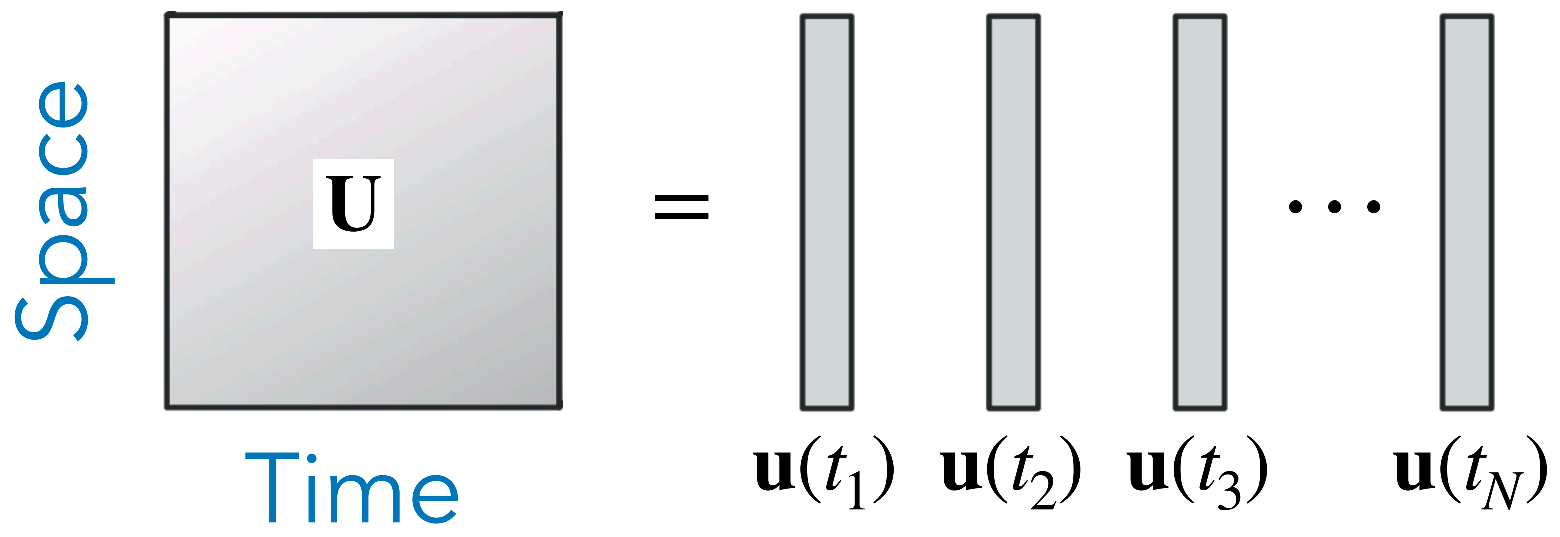
Dan Amrhein, Dafydd Stephenson
National Center for Atmospheric Research

LuAnne Thompson, Noah Rosenberg
University of Washington

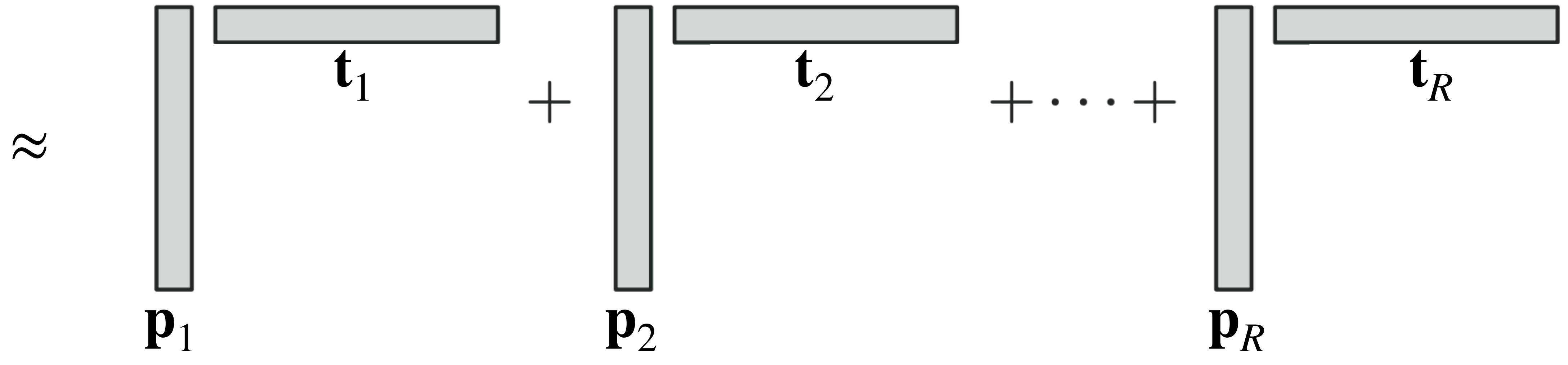
What are the **dominant patterns and pathways** by which the atmosphere drives **ocean variability**?

The leading **EOF** answers the question:

What atmospheric pattern accounts for the greatest fraction of total atmospheric variance?

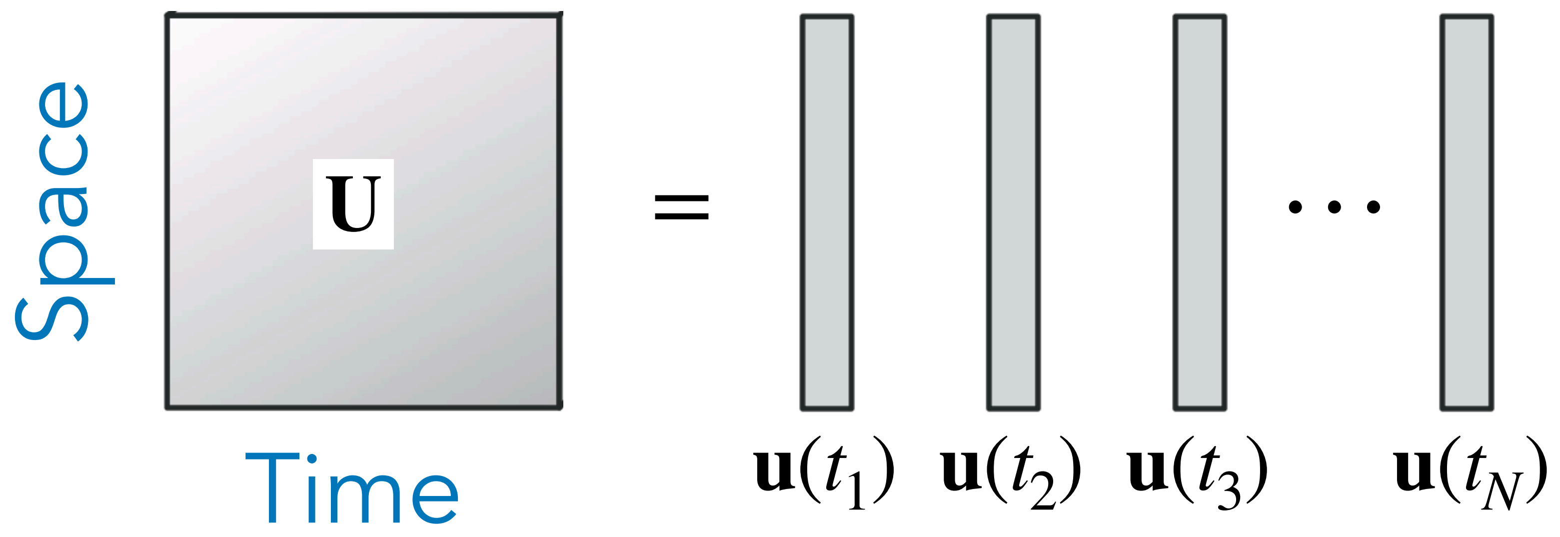


Concatenate your favorite atmospheric variable into a data matrix…

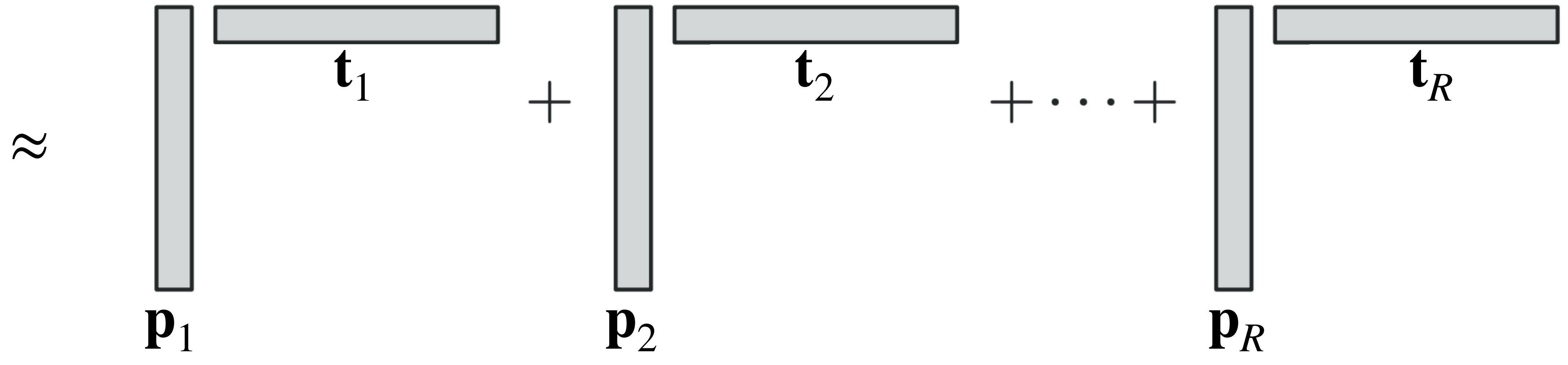


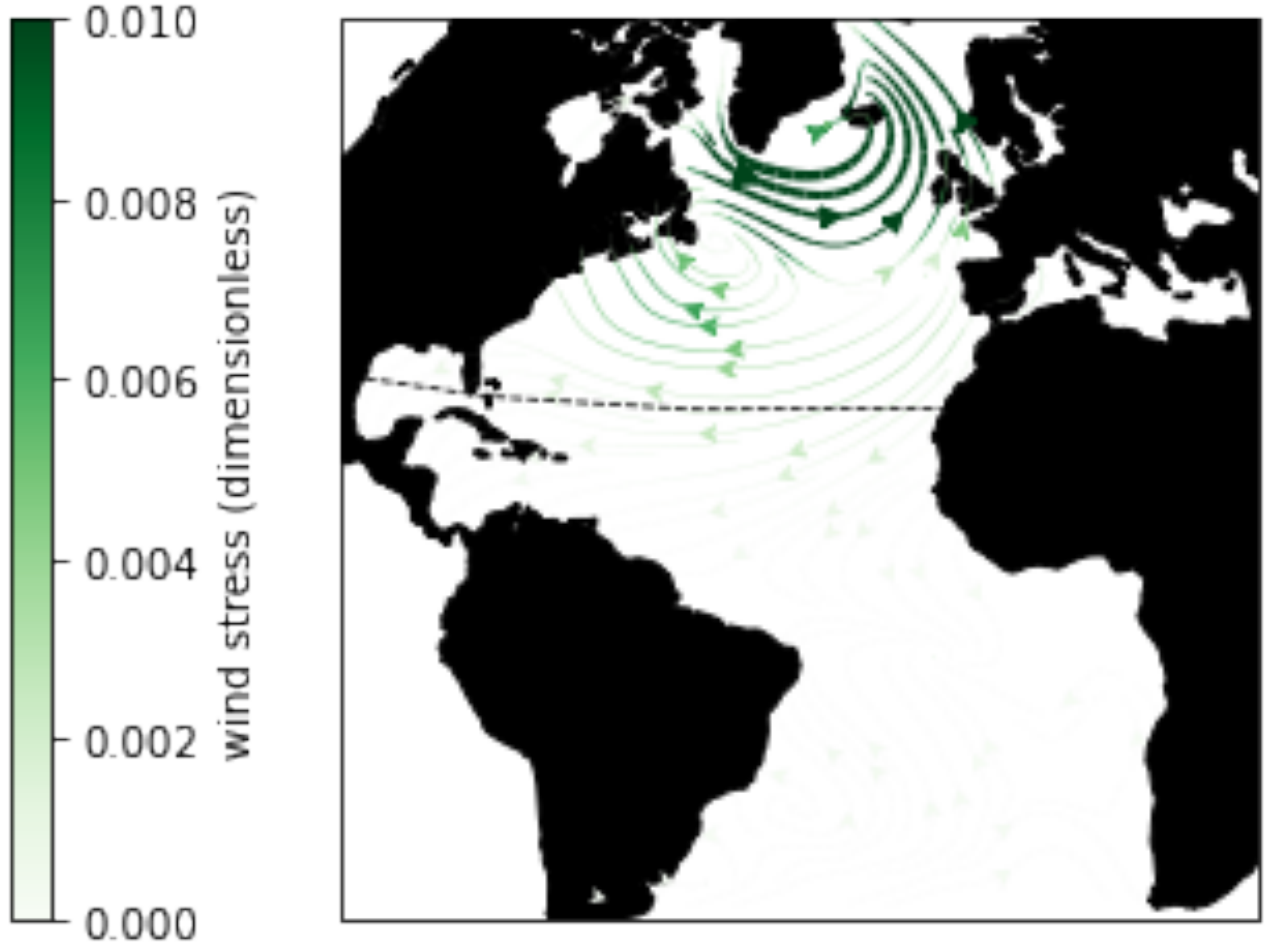
The leading **EOF** answers the question:

What atmospheric pattern accounts for the greatest fraction of total atmospheric variance?



Concatenate your favorite atmospheric variable into a data matrix…





The leading EOF of wind stress in the ECCO v4r4 state estimate.

The leading **SO** (stochastic optimal; *Farrell and Ioannou 1993, 1996; Kleeman and Moore, 1997*) answers the question:

What (hypothetical) atmospheric pattern most efficiently excites variance in the ocean?

Ocean model adjoint sensitivities diagnose dominant drivers

$$\mathbf{s} = \frac{\partial x}{\partial \mathbf{u}}$$

"Quantity of interest"

Any function of the model state
(e.g., AMOC strength at 26N)

"Controls"

Vector in time and space of ocean model inputs that can change x
(e.g., atmospheric fluxes)

Adjoint sensitivity

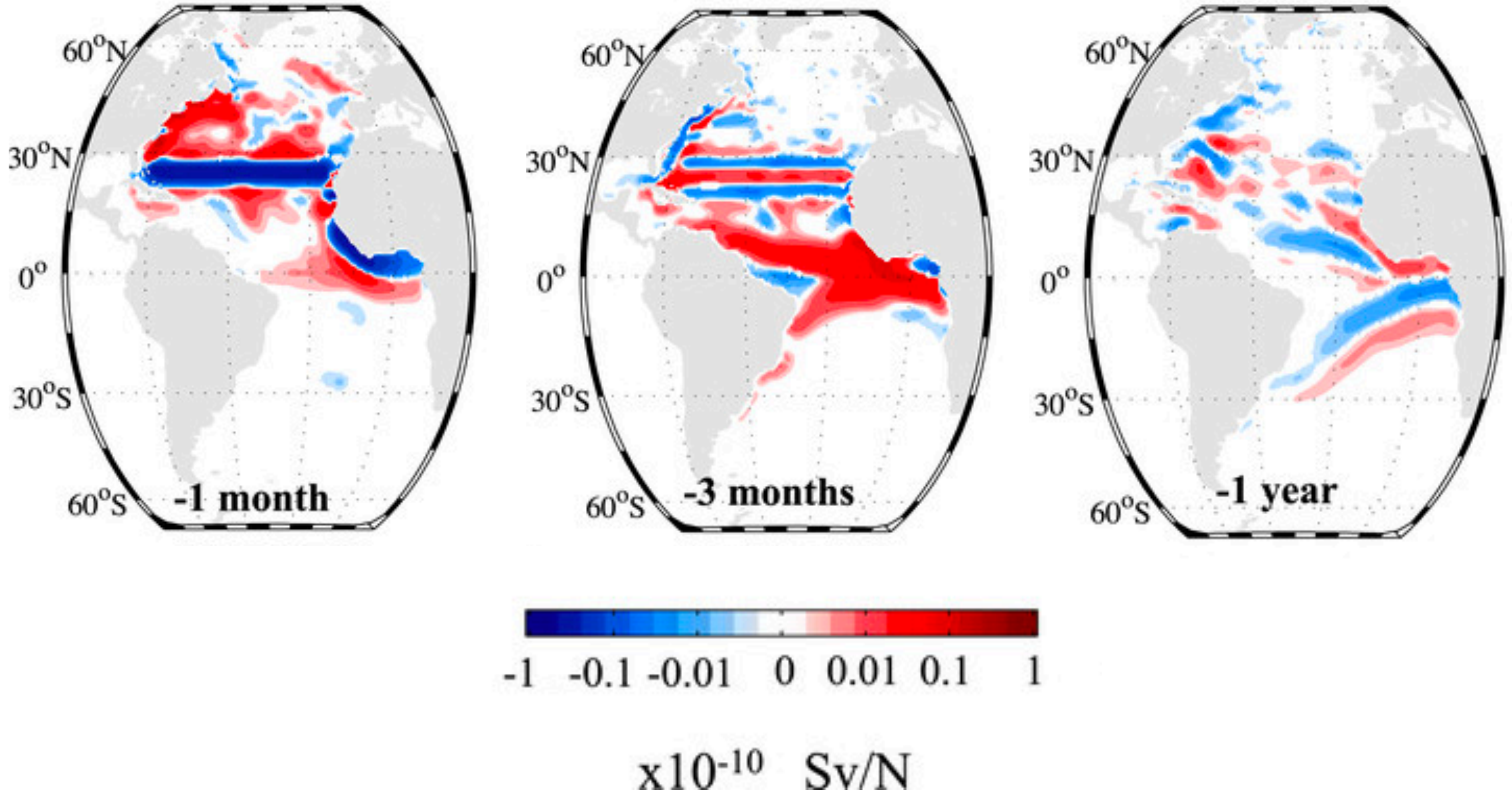
How much will changing $\mathbf{u}$ change x?
(A *locally linear* estimate)

Ocean model adjoint sensitivities diagnose dominant drivers

$$\mathbf{s} = \frac{\partial x}{\partial \mathbf{u}}$$

AMOC strength
@ 26N in January

Zonal wind stress



Pillar et al. 2016

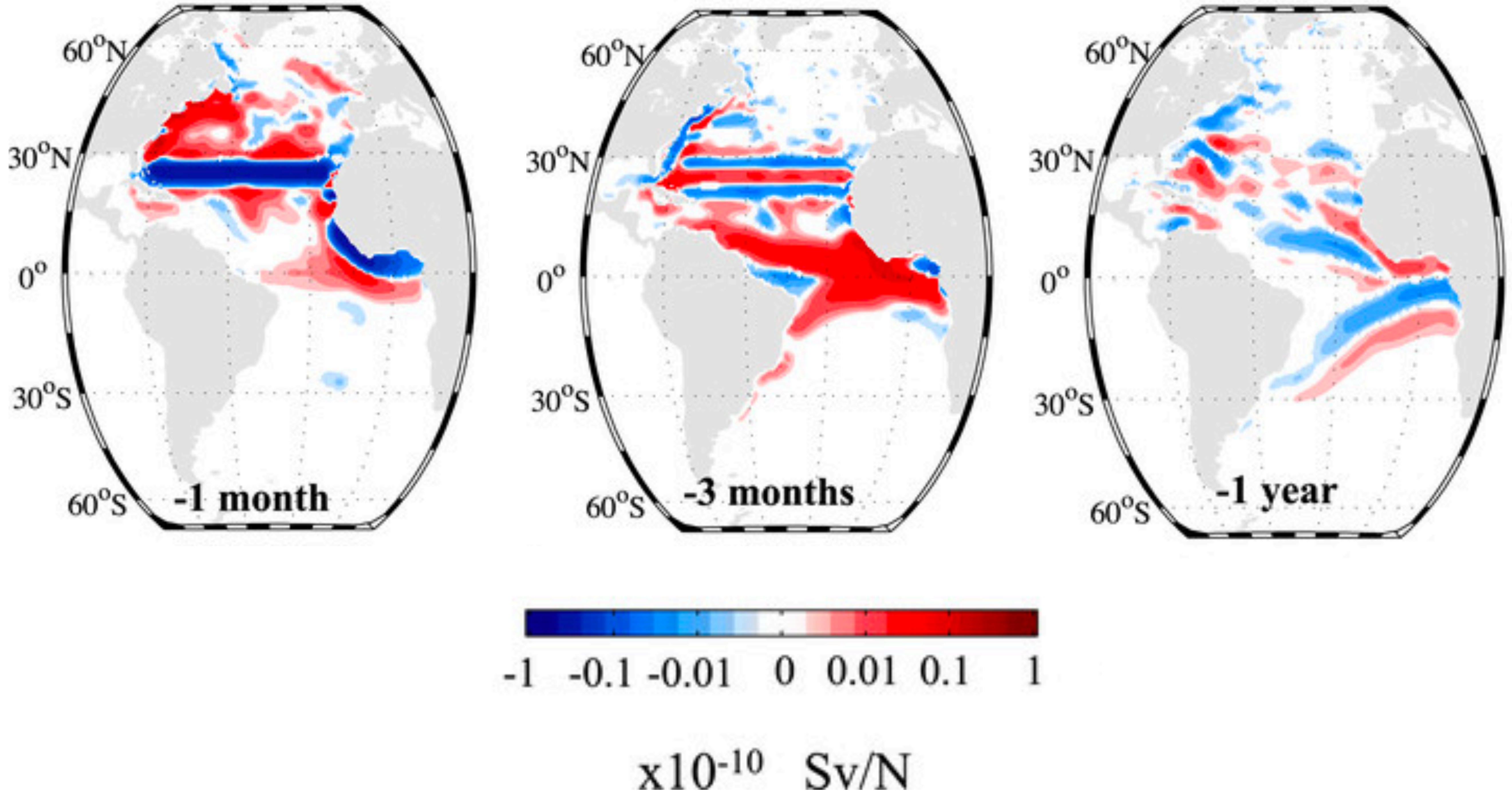
Also Heimbach and Wunsch 2011; Jones et al. 2018; Kostov et al. 2019, 2021; Fukumori et al. 2021; Stephenson and Sevellec 2020, 2021

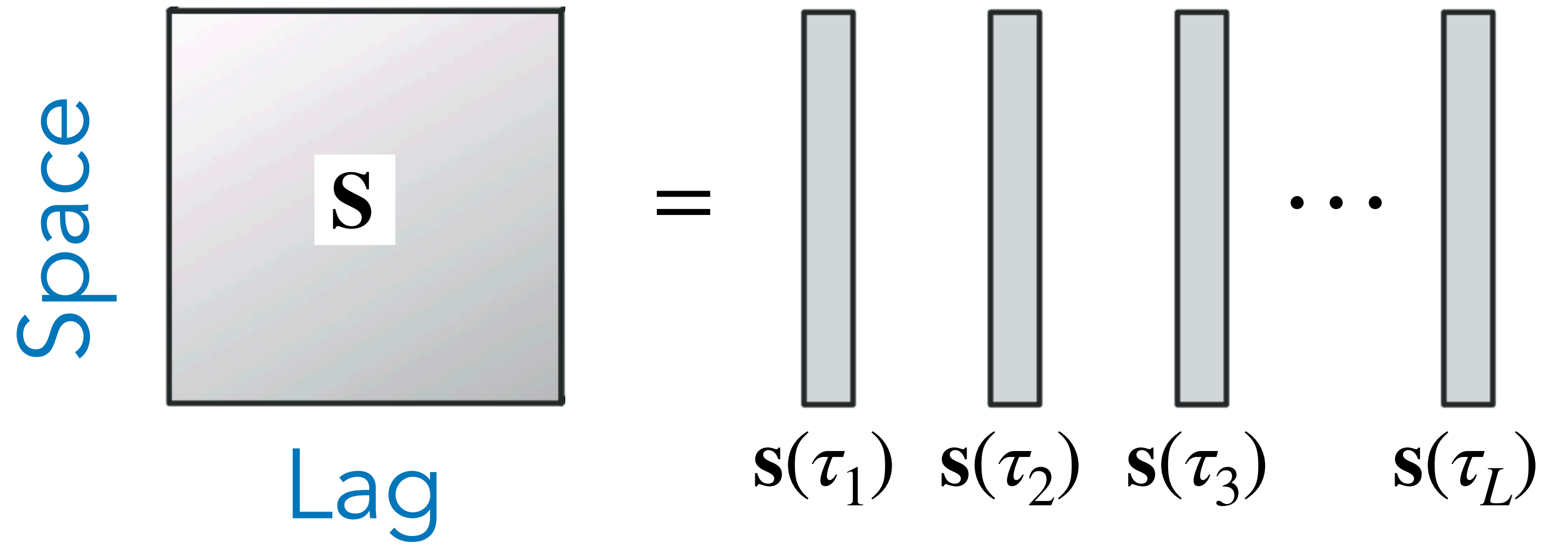
Ocean model adjoint sensitivities diagnose dominant drivers

$$\mathbf{s} = \frac{\partial x}{\partial \mathbf{u}}$$

AMOC strength @ 26N in January (∂x)

Zonal wind stress ($\partial \mathbf{u}$)



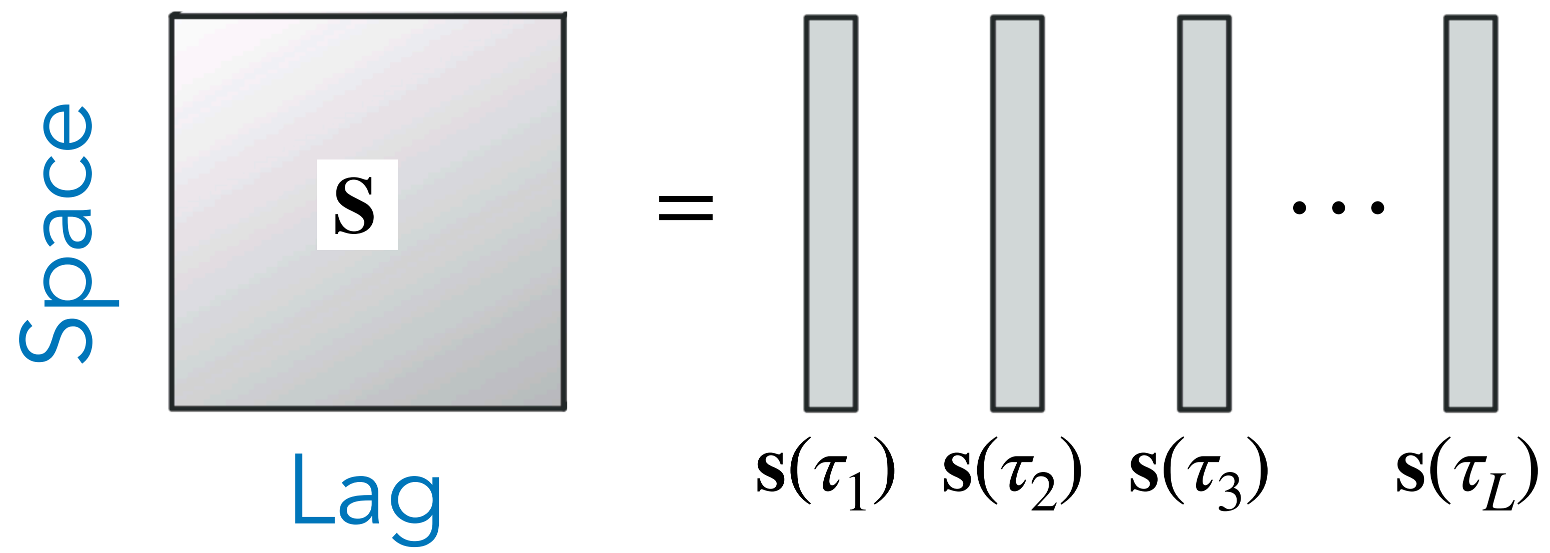


Pillar et al. 2016

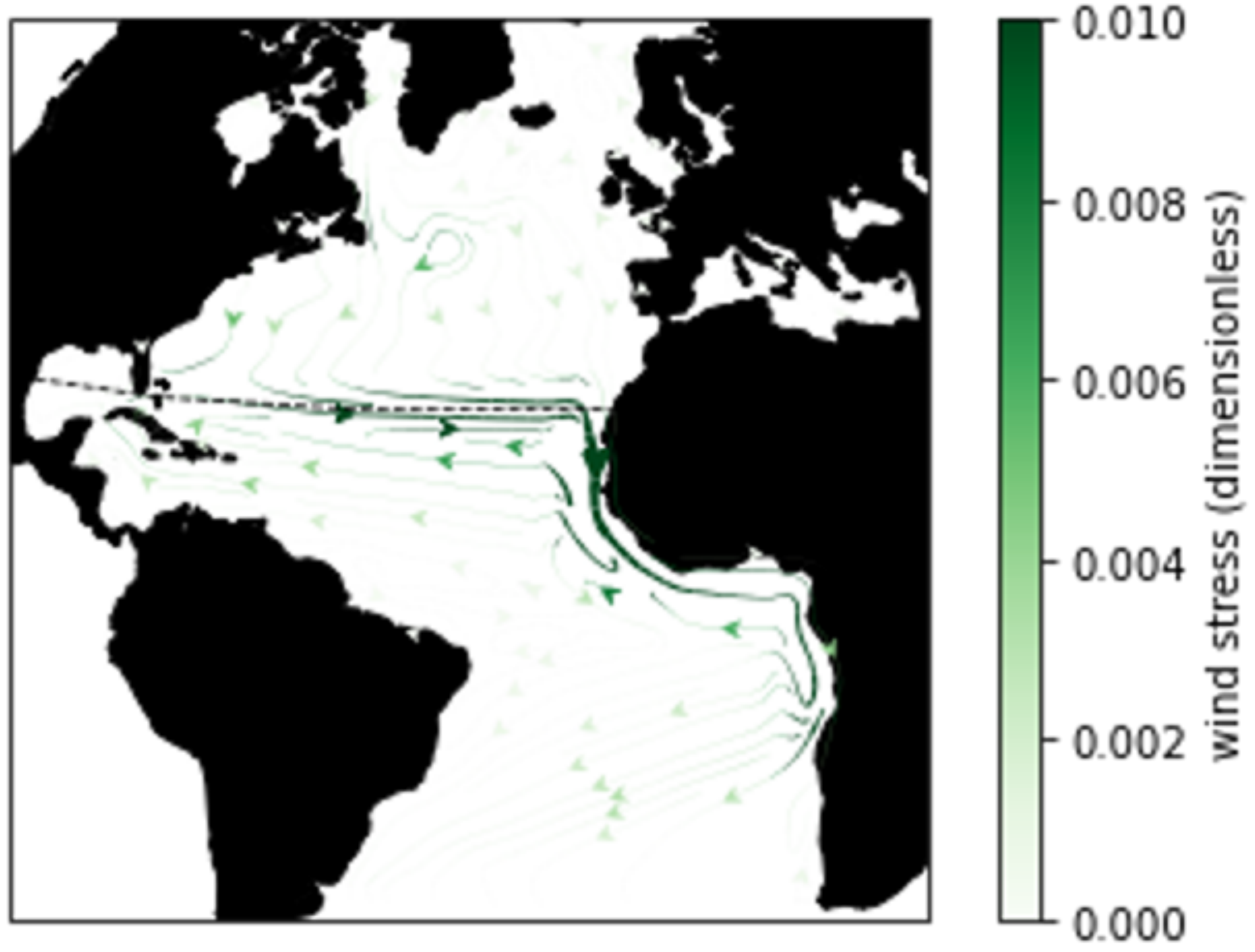
Also Heimbach and Wunsch 2011; Jones et al. 2018; Kostov et al. 2019, 2021; Fukumori et al. 2021; Stephenson and Sevellec 2020, 2021

The leading **SO** (stochastic optimal; *Farrell and Ioannou 1993, 1996; Kleeman and Moore, 1997*) answers the question:

What (hypothetical) spatial pattern most efficiently excites variance in the ocean?



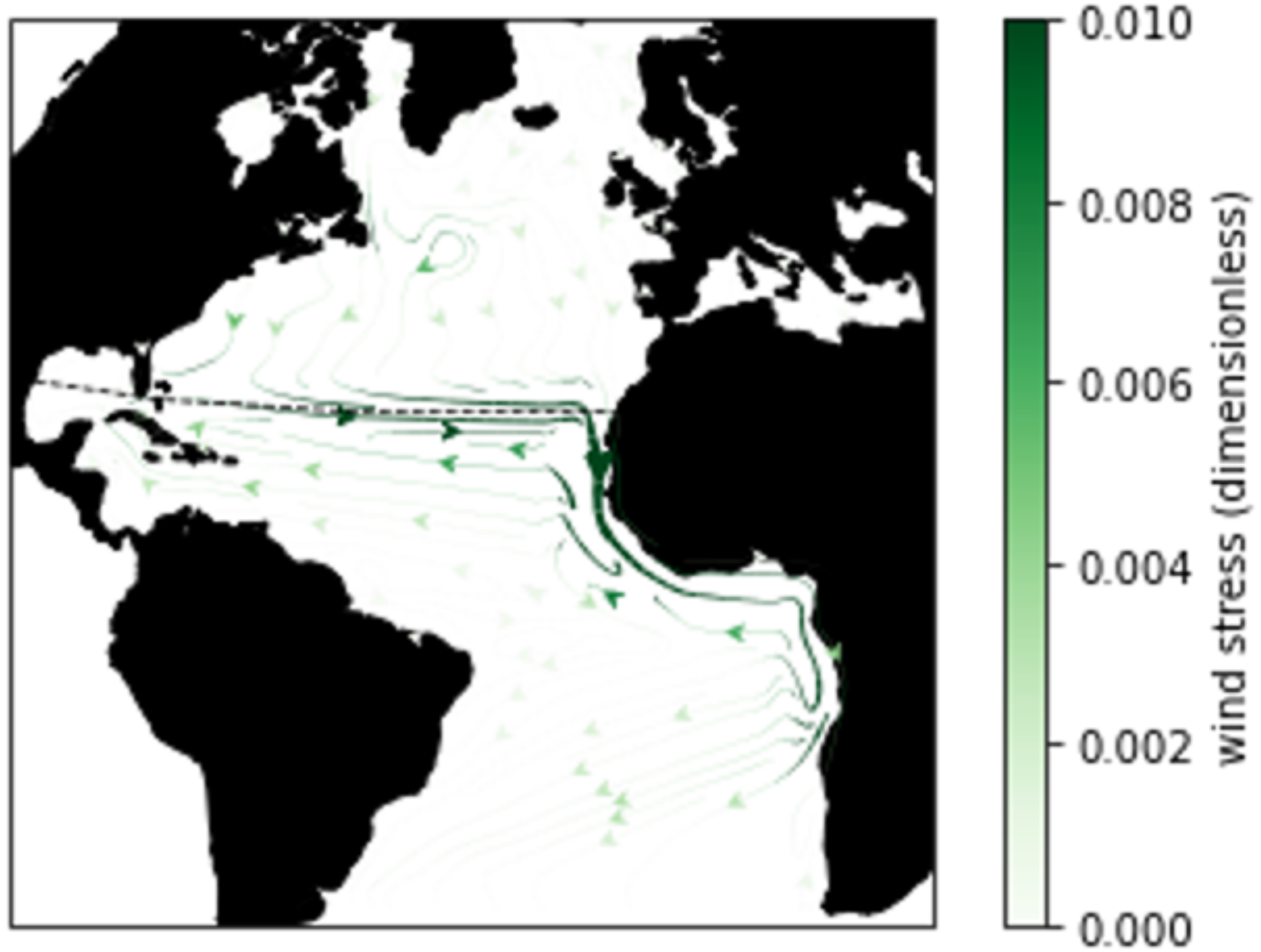
$$\mathbf{Z} = \mathbf{S}\mathbf{S}^\top$$



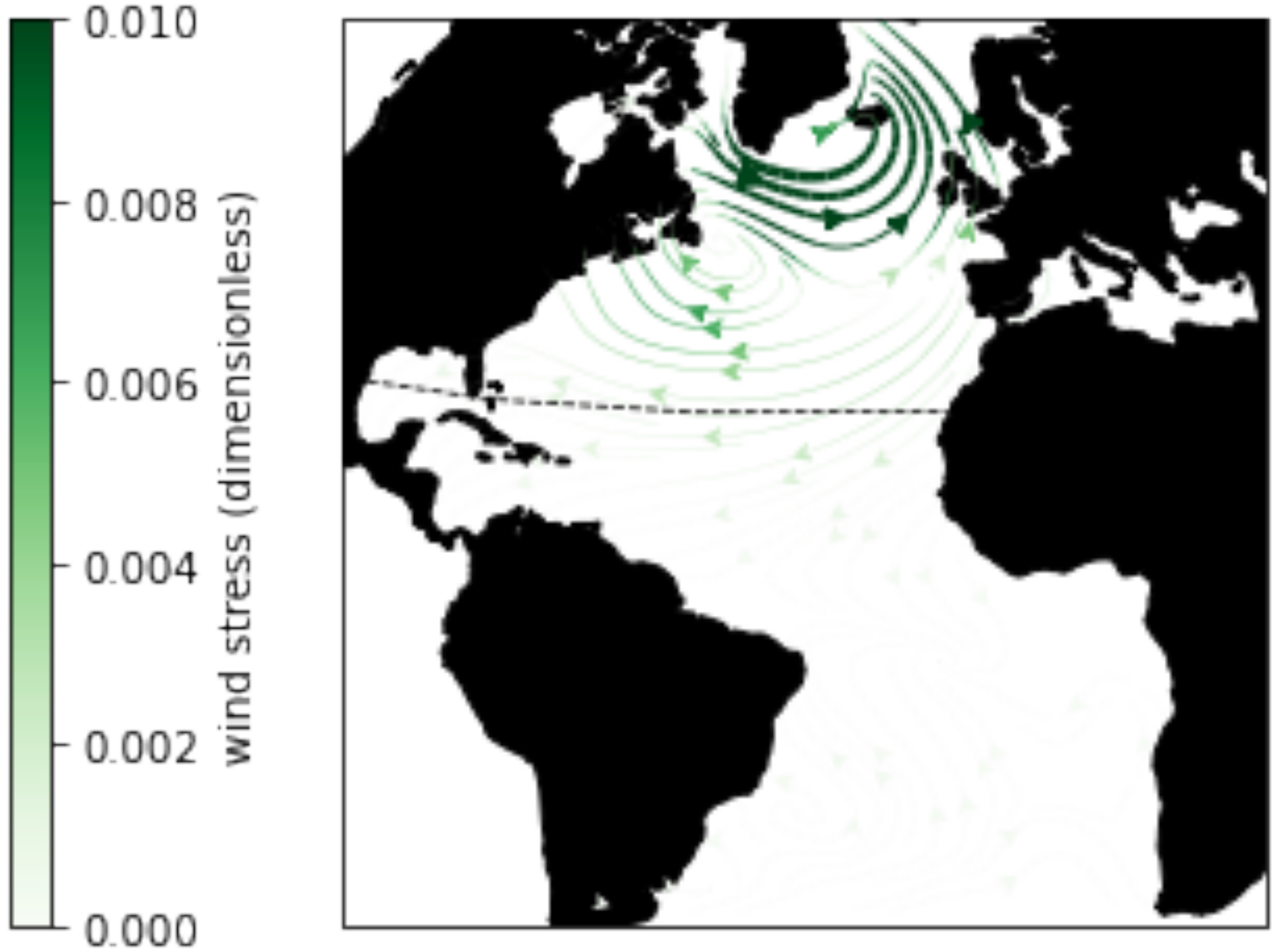
Stochastic optimals are the eigenvectors of $\mathbf{Z}$, here computed for wind stress in the ECCO v4r4 state estimate.

An interpretive quandary

What the ocean “wants”



What the ocean “gets”

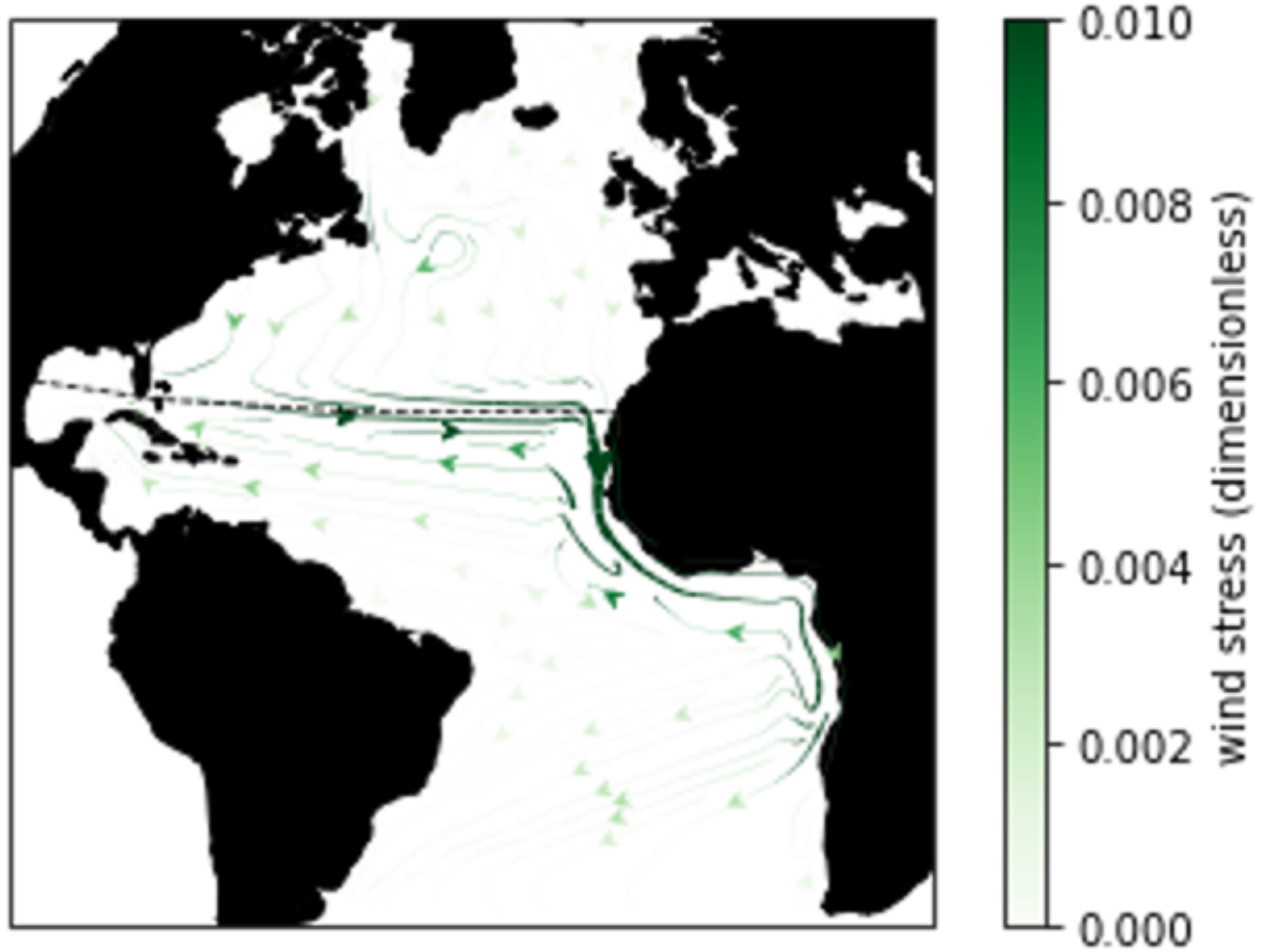


Is the leading EOF the **leading driver** of variability in this ocean quantity?

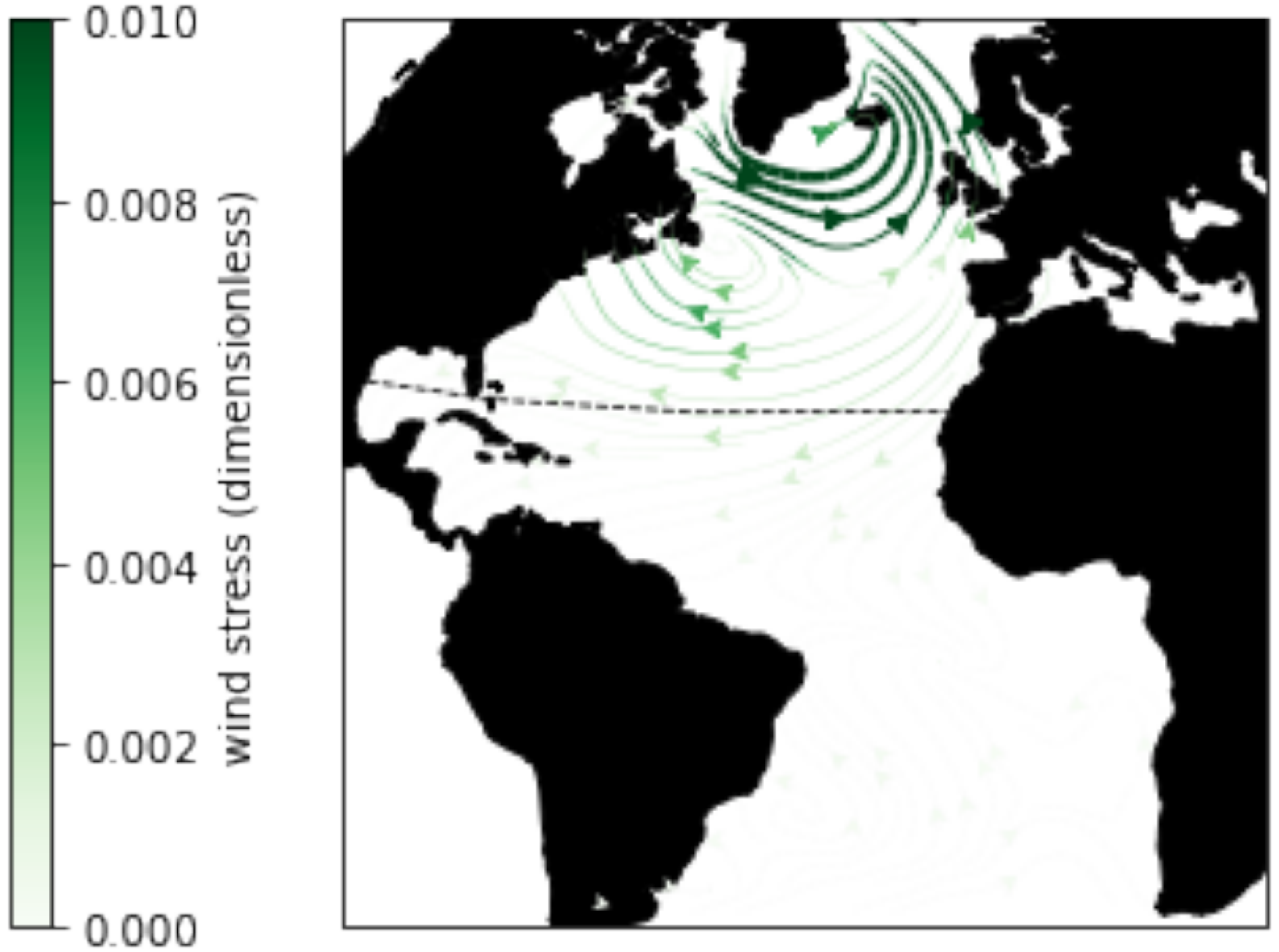
Is the leading stochastic optimal really the most important **mechanism** for changing the ocean?

An interpretive quandary

What the ocean “wants”



What the ocean “gets”



?

Is the leading EOF the **leading driver** of variability in this ocean quantity?

Is the leading stochastic optimal really the most important **mechanism** for changing the ocean?

Our goal is to derive atmospheric patterns that **maximize contributions to ocean variability.**

The math slide! Deriving dynamics-weighted principal components

$$\mathbf{s} = \frac{\partial x}{\partial \mathbf{u}}$$

Definition of adjoint sensitivity

The math slide! Deriving dynamics-weighted principal components

$$\mathbf{s} = \frac{\partial x}{\partial \mathbf{u}}$$

Definition of adjoint sensitivity

$$\delta x(t) \approx \sum_{i=1}^{N_\tau} \mathbf{s}(\tau_i)^\top \delta \mathbf{u}(t - \tau_i)$$

Modifying Fukumori et al. 2015

The math slide! Deriving dynamics-weighted principal components

$$\mathbf{s} = \frac{\partial x}{\partial \mathbf{u}}$$

Definition of adjoint sensitivity

$$\delta x(t) \approx \sum_{i=1}^{N_\tau} \mathbf{s}(\tau_i)^\top \delta \mathbf{u}(t - \tau_i)$$

Modifying Fukumori et al. 2015

$$\sigma_\Sigma^2 = \left\langle \left(\delta x(t)\right)^2 \right\rangle$$

The variance of the quantity of interest

The math slide! Deriving dynamics-weighted principal components

$$\mathbf{s} = \frac{\partial x}{\partial \mathbf{u}}$$

Definition of adjoint sensitivity

$$\delta x(t) \approx \sum_{i=1}^{N_\tau} \mathbf{s}(\tau_i)^\top \delta \mathbf{u}(t-\tau_i)$$

Modifying Fukumori et al. 2015

$$\sigma_\Sigma^2 = \left\langle \left(\delta x(t)\right)^2 \right\rangle$$

The variance of the quantity of interest

$$= \sum_{i=1}^{N_\tau} \sum_{j=1}^{N_\tau} \mathbf{s}(\tau_i)^\top \left\langle \delta \mathbf{u}(t-\tau_i) \delta \mathbf{u}^\top (t-\tau_j) \right\rangle \mathbf{s}(\tau_j)$$

Substitution gets a bit sticky…

$$= \mathbf{tr}(\mathbf{C}\mathbf{Z})$$

Atmospheric spatial covariance

$$\mathbf{Z} = \mathbf{S}\mathbf{S}^\top$$

…but is simplified by two assumptions (see also Kleeman and Moore, 1997):

1. Flux covariances are separable in space and time
2. Sensitivities are stationary

The math slide! Deriving dynamics-weighted principal components

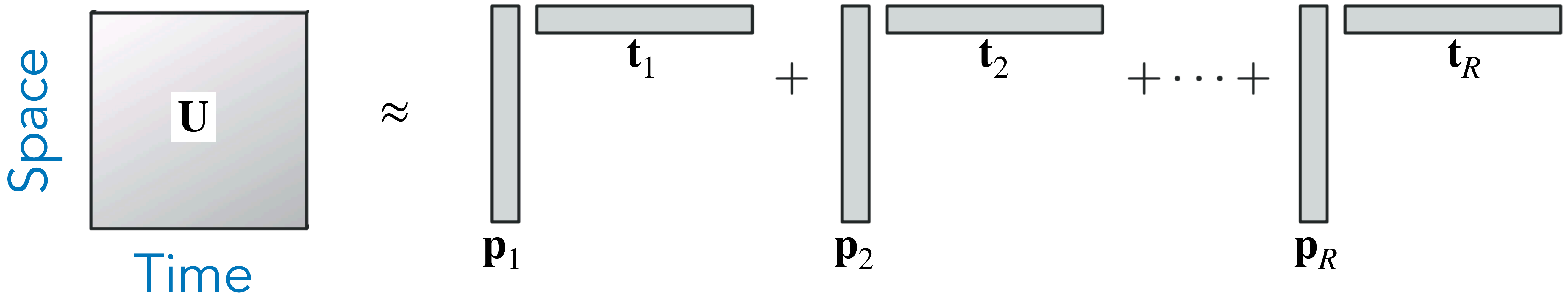
$$\sigma_\Sigma^2 = \mathbf{tr}(\mathbf{CZ})$$

$$\mathbf{U} = \sum \mathbf{p}_k \mathbf{t}_k^\top$$

$$\sigma_\Sigma^2 = \sum \sigma_k^2$$

Our requirements:

1. An EOF-like decomposition
2. Contributions to ocean variance that add (no cross terms)



The math slide! Deriving dynamics-weighted principal components

$$\sigma_\Sigma^2 = \mathrm{\mathbf{tr}}(\mathbf{CZ})$$

$$\mathbf{U} = \sum \mathbf{p}_k \mathbf{t}_k^\top$$

$$\sigma_\Sigma^2 = \sum \sigma_k^2$$

Our requirements:

1. An EOF-like decomposition
2. Contributions to ocean variance that add (no cross terms)

Contributions to QoI variance

$$\mathbf{S}^\top \mathbf{U} = \mathbf{L\Gamma T}^\top$$

$$\mathbf{P} = \mathbf{UT}$$

Spatial patterns ranked by their contribution to ocean QoI variance

…yields an SVD optimization problem!

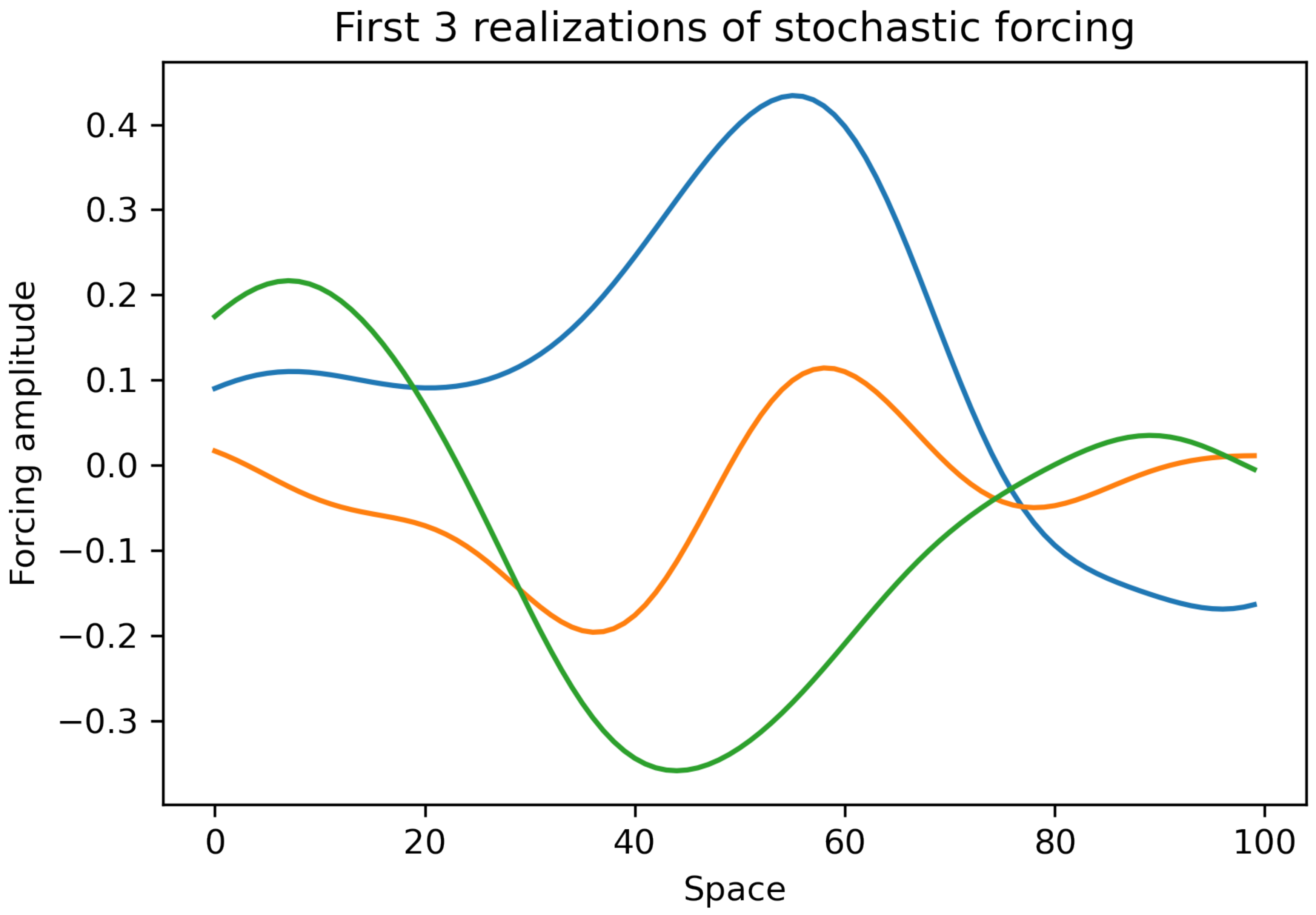
Amounts to computing principal components weighted by adjoint sensitivities.

EOF-like, but singular values are ocean QoI variance rather than atmospheric variance.

Patterns are orthogonal in time, but not space.

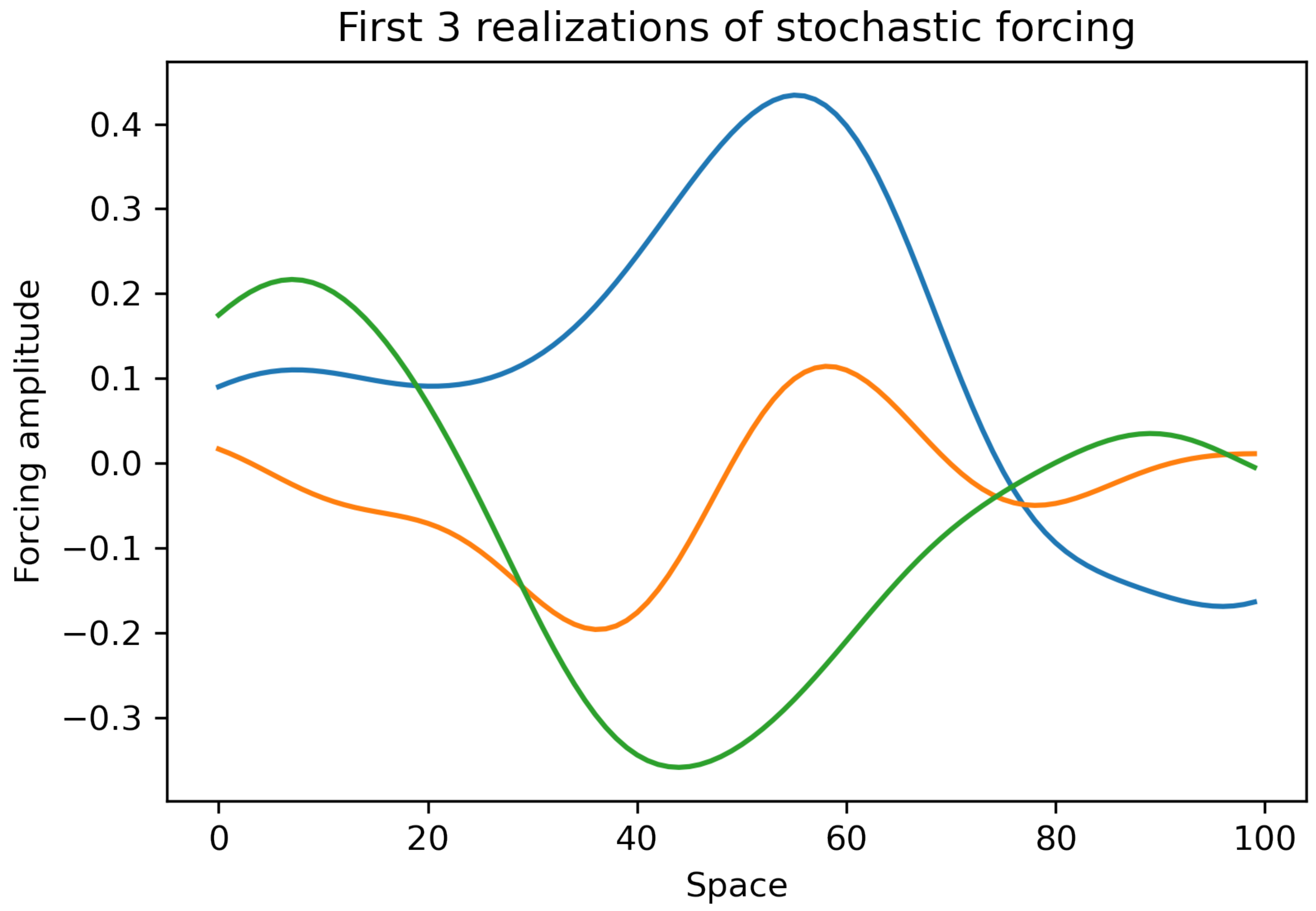
Recovers EOFs and SOs for limit cases.

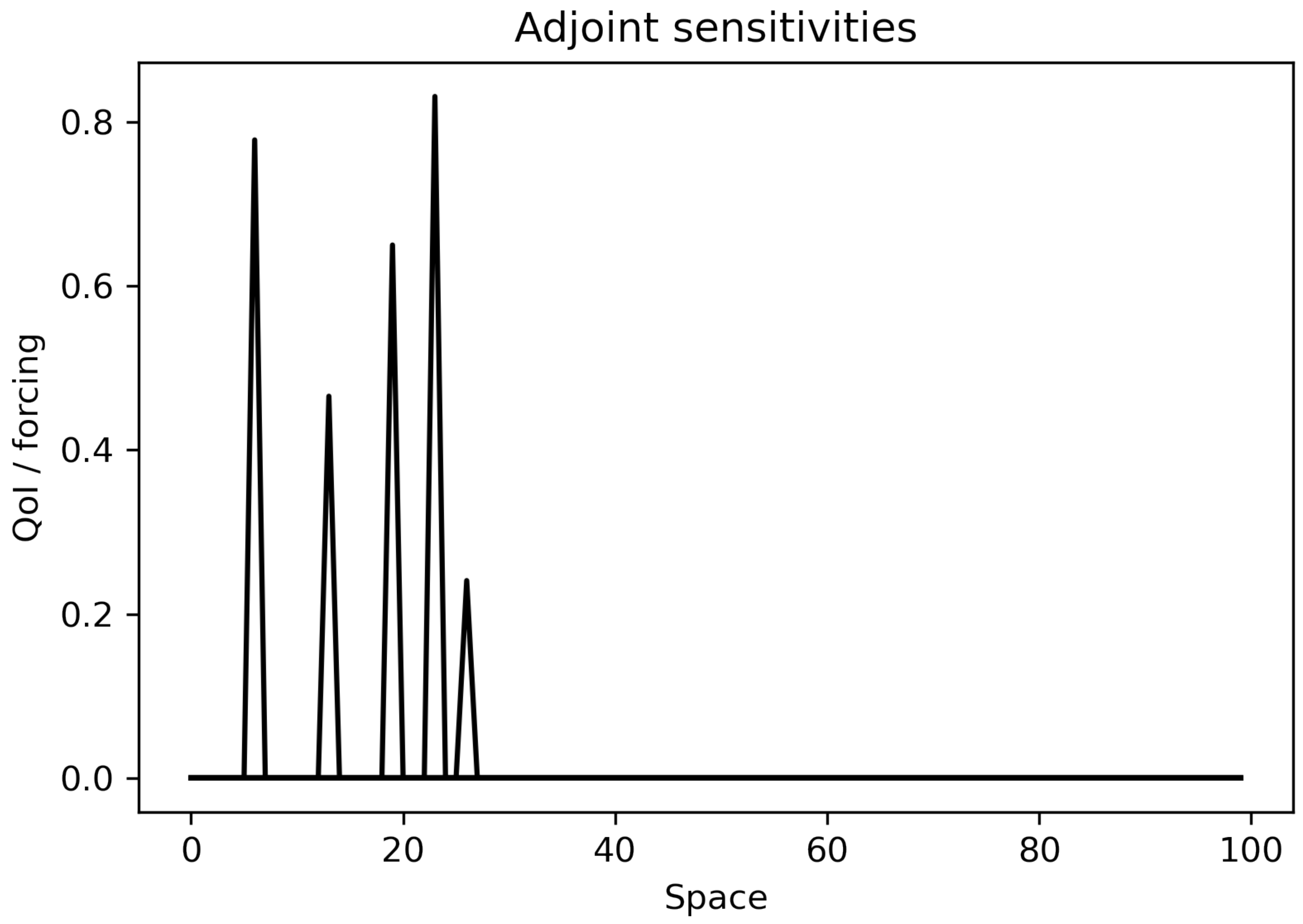
Demonstration in a (very) simple system



Consider a 1-dimensional system with stochastic forcing that is **smooth in space** and Gaussian **white noise in time**.

Demonstration in a (very) simple system

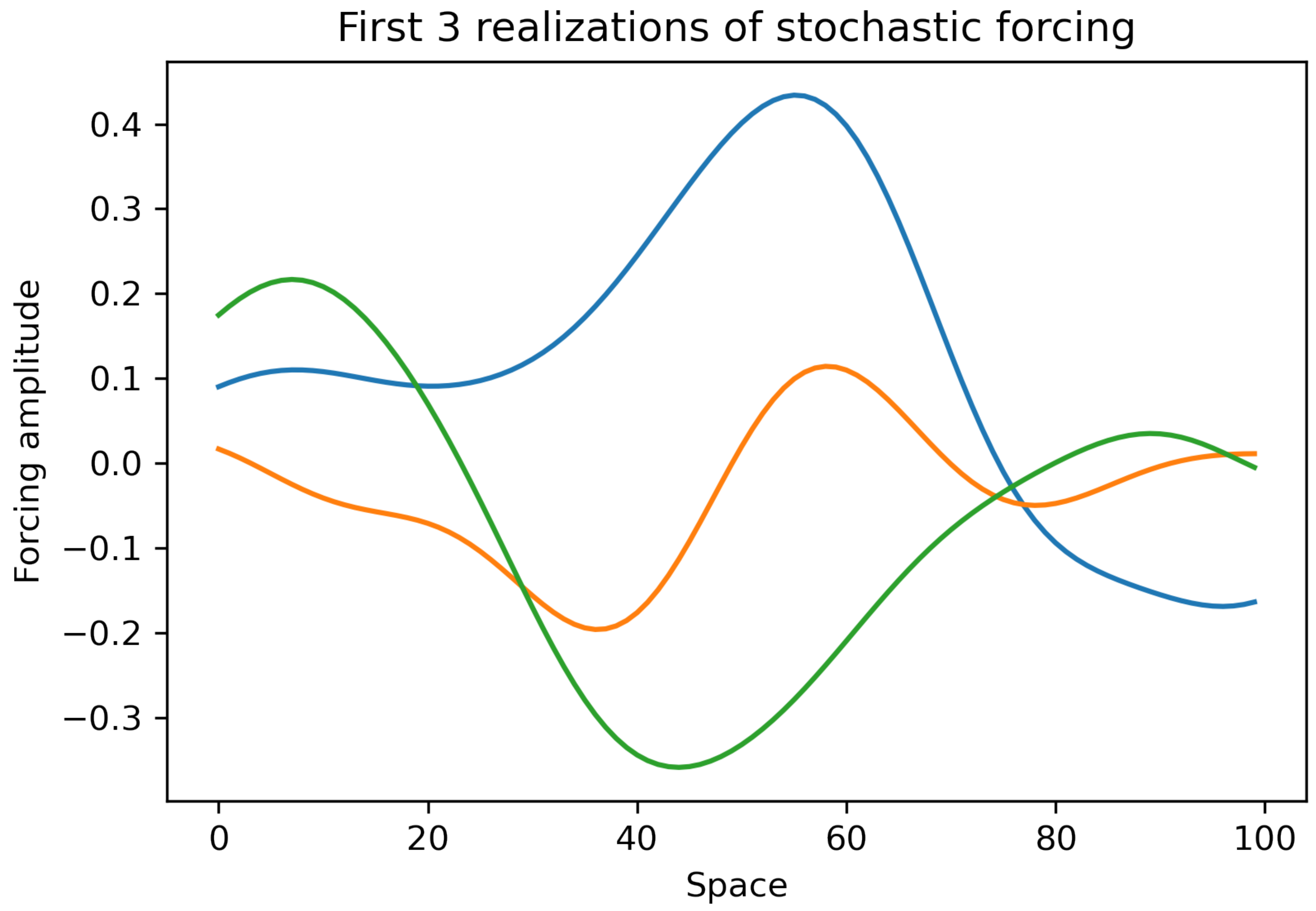


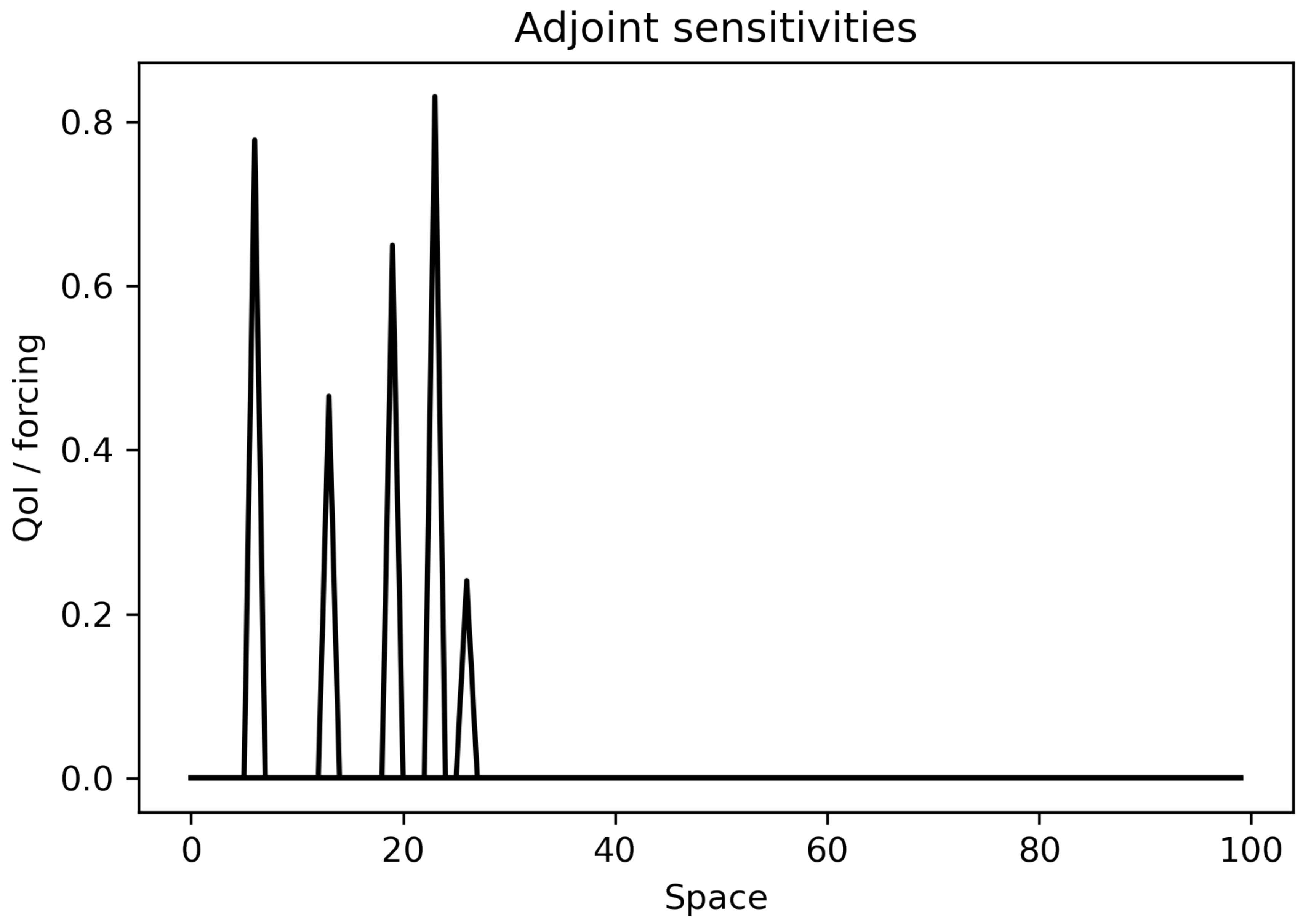


Consider a 1-dimensional system with stochastic forcing that is **smooth in space** and Gaussian **white noise in time**.

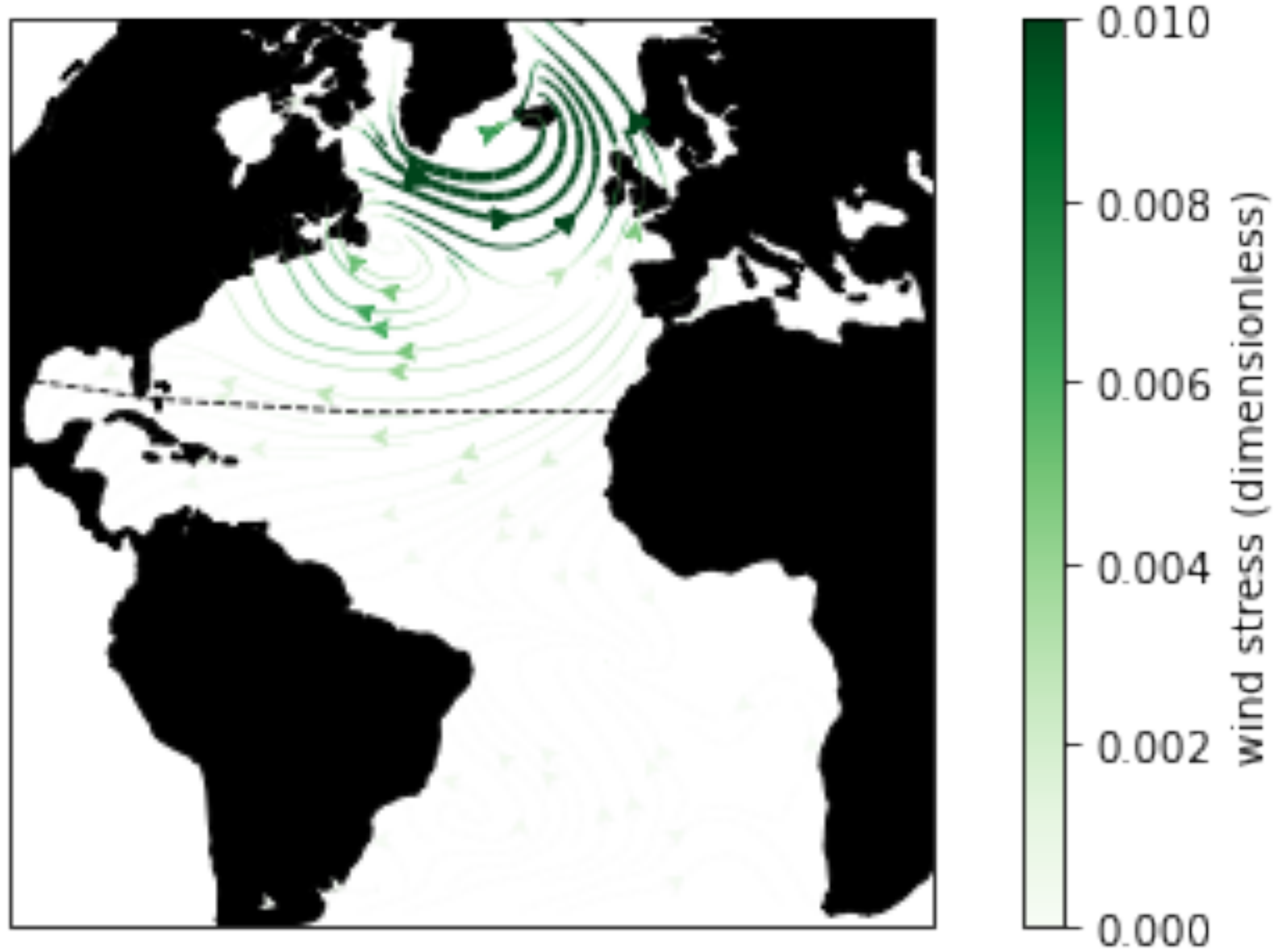
..and adjoint sensitivities of a hypothetical ocean QoI that have **shorter length scales** and are **localized in space**.

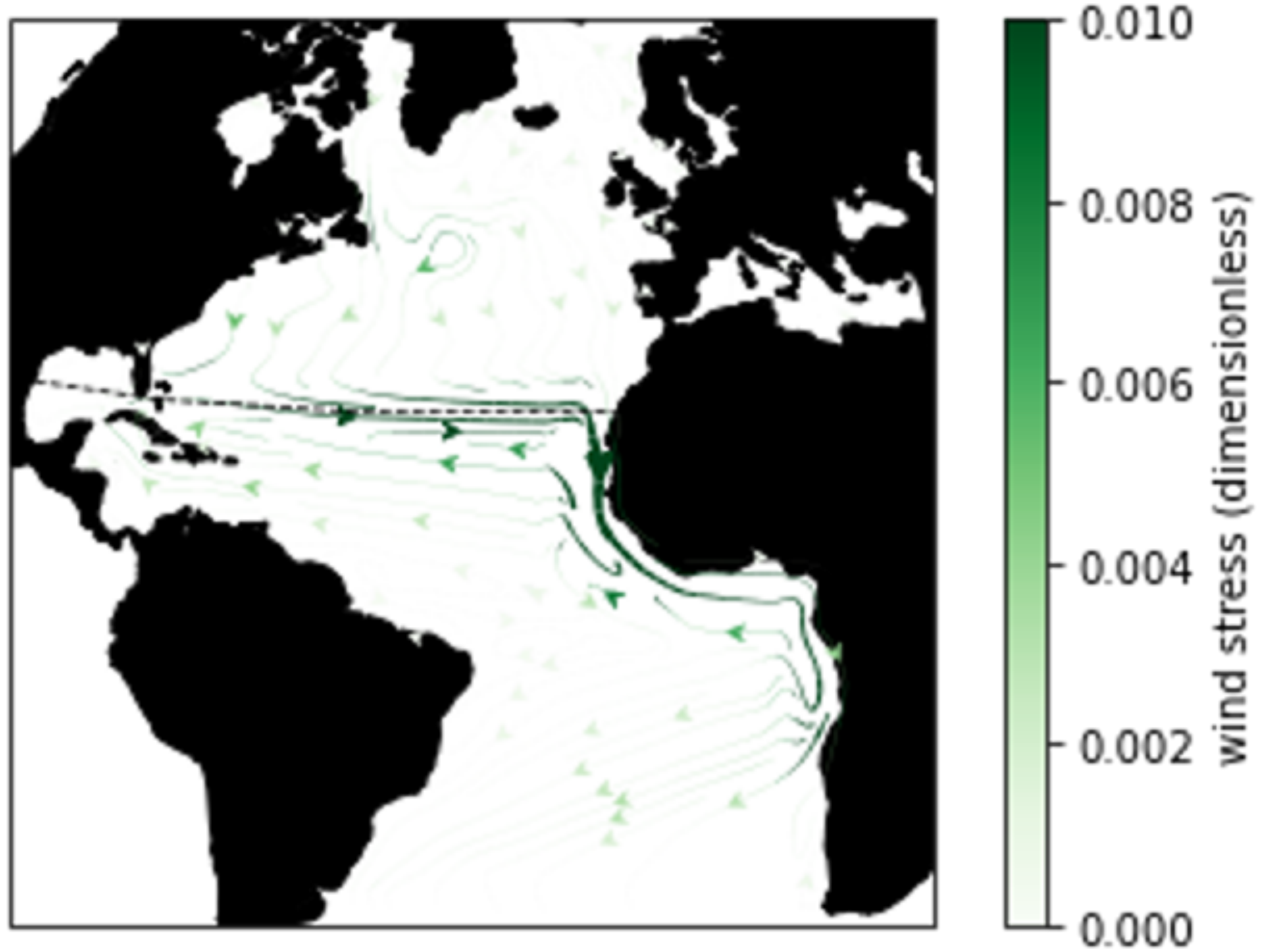
Demonstration in a (very) simple system



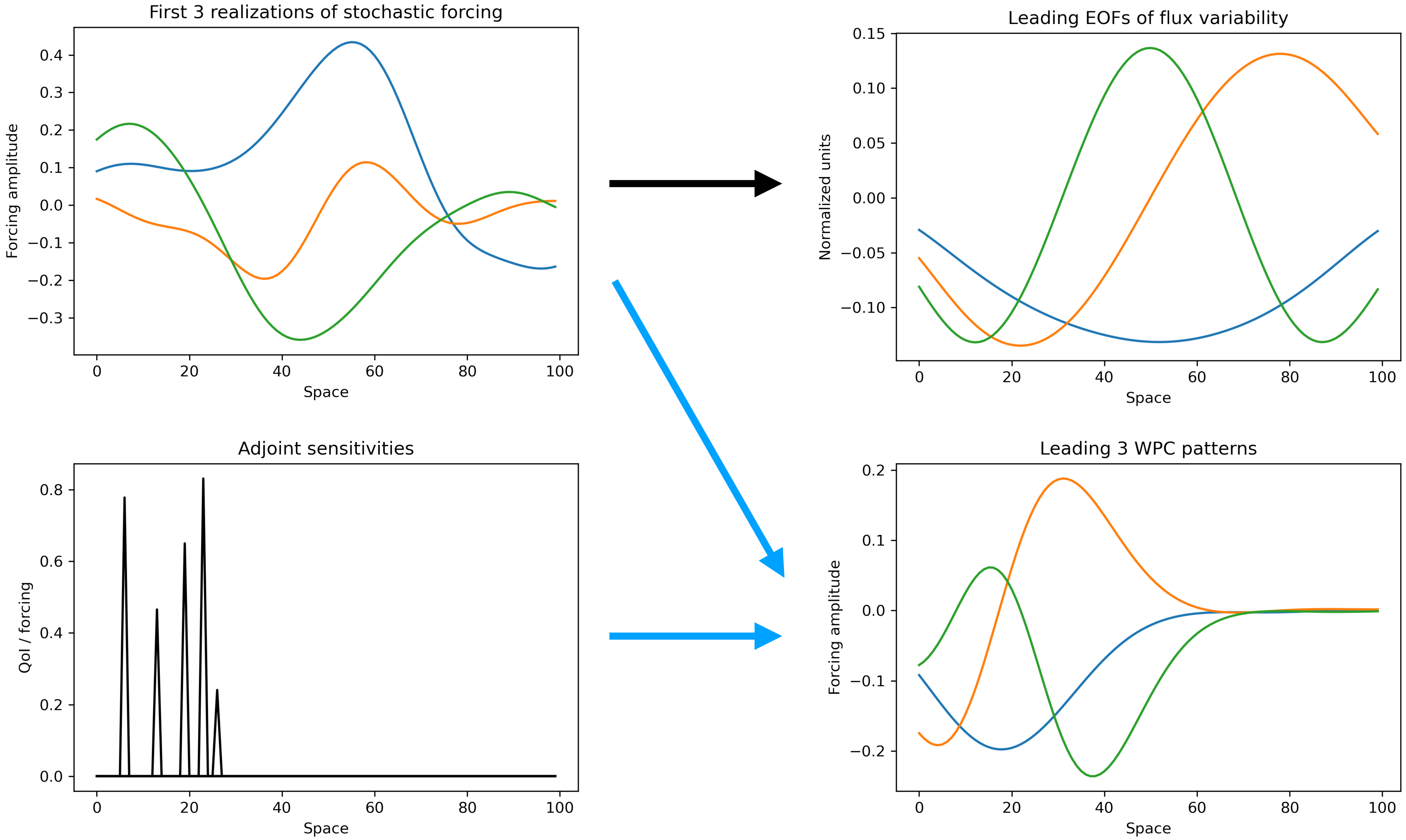


~



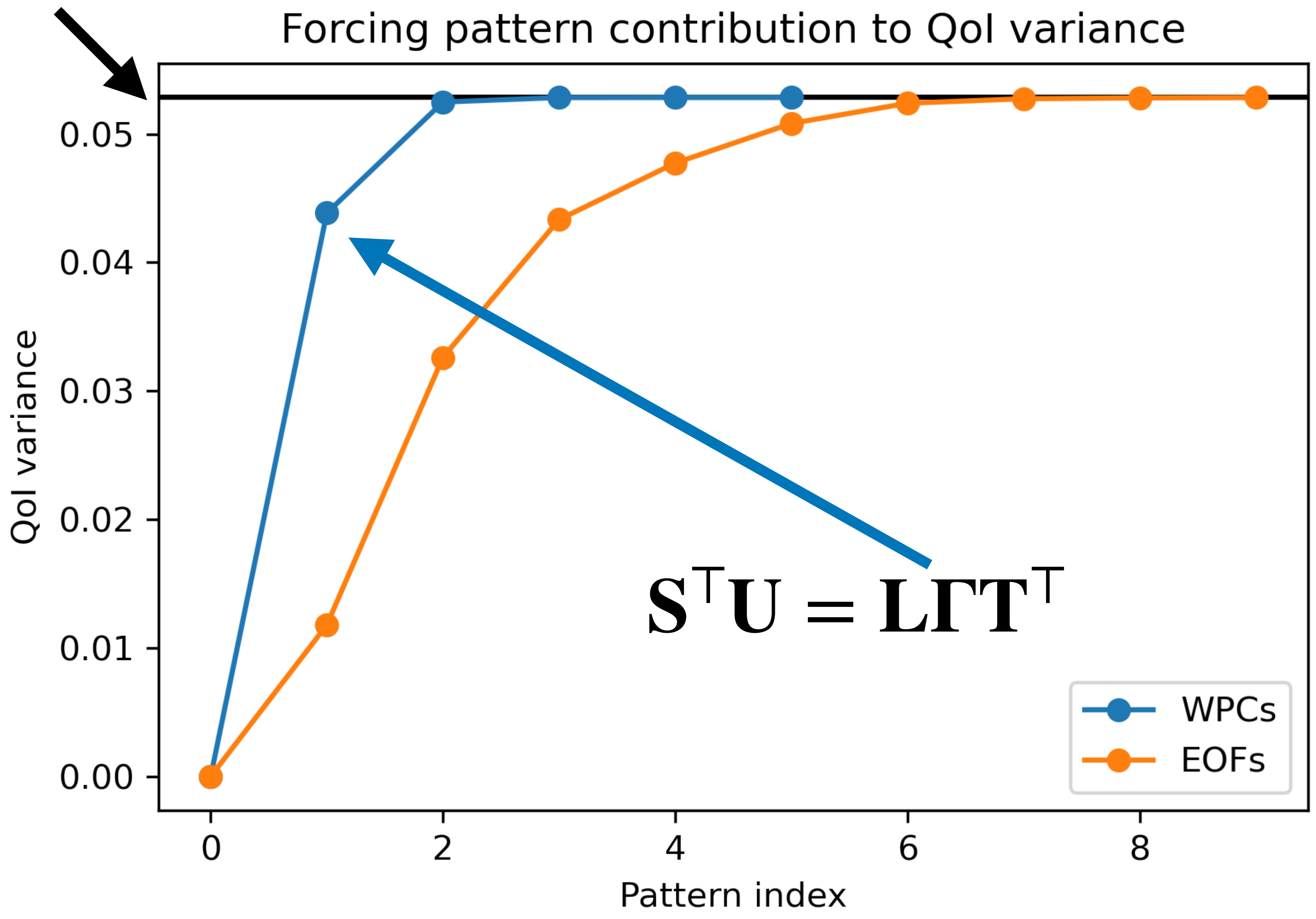


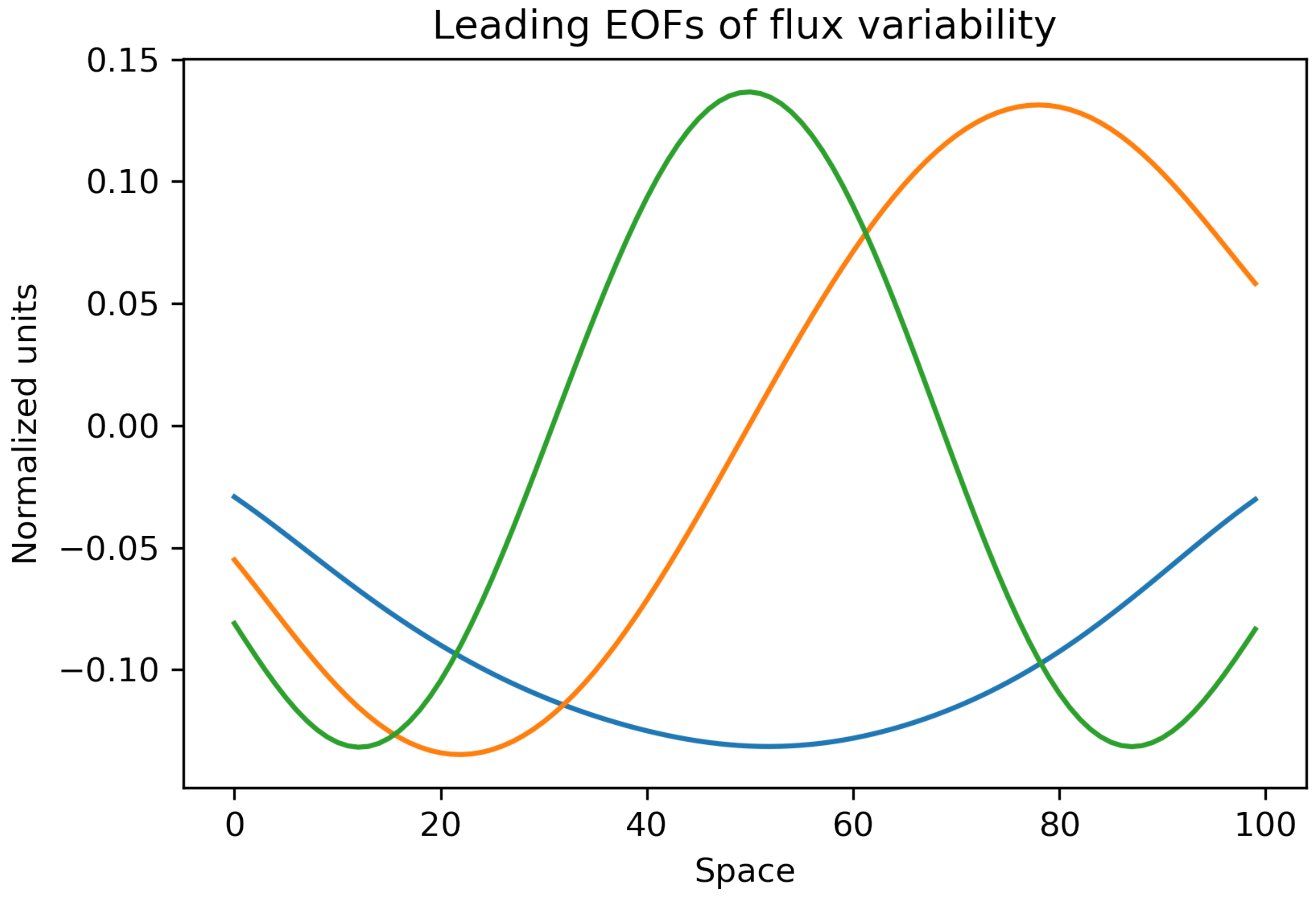
Demonstration in a (very) simple system

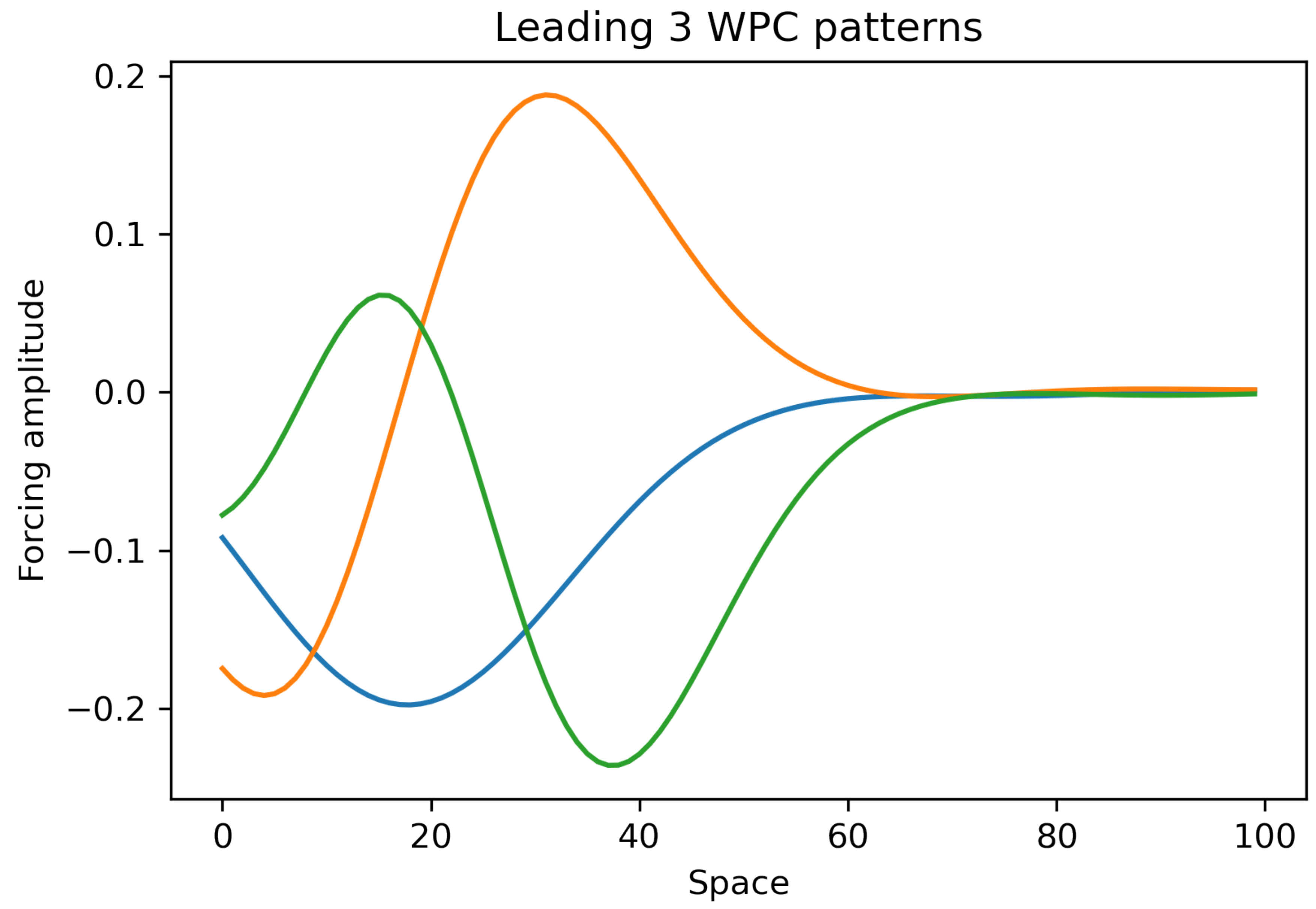


Demonstration in a (very) simple system

Total QoI variance







WPC patterns outperform EOFs at driving QoI variance.

Conclusions

Adjoints tell us what the **ocean "wants" from the atmosphere.**

Atmospheric EOFs describe **dominant atmospheric patterns.**

Dynamics-weighted principal components identify ***atmospheric*** structures that dominate ***ocean*** **variability.**

We argue that these patterns are a useful complement to the regions of "potential" influence for variance budgets and observing system design.

We are evaluating whether similar approaches for "smoothing" sensitivities improves their utility across multiple ocean models.

Dafydd's talk will give this a spin in the MITgcm and ECCO!