Uncertainty Quantification of Ocean Driven Melting Under the Pine Island Ice Shelf

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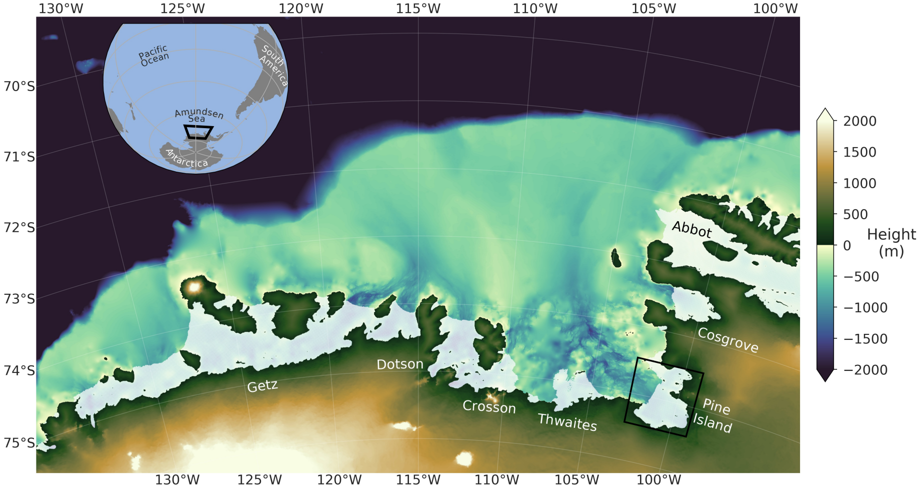
Cooperative Institute for Research in Environmental Sciences
UNIVERSITY OF COLORADO BOULDER and NOAA

The University of Texas at Austin
Oden Institute for Computational Engineering and Sciences

The Amundsen Sea, West Antarctica



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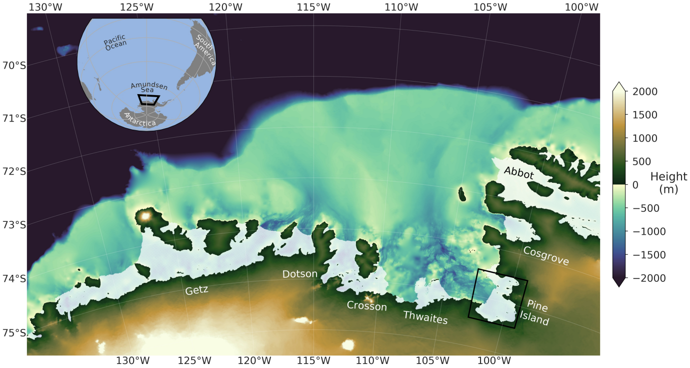

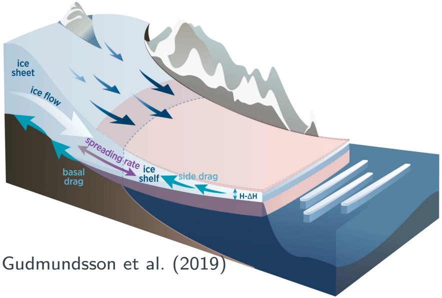

Gudmundsson et al. (2019)

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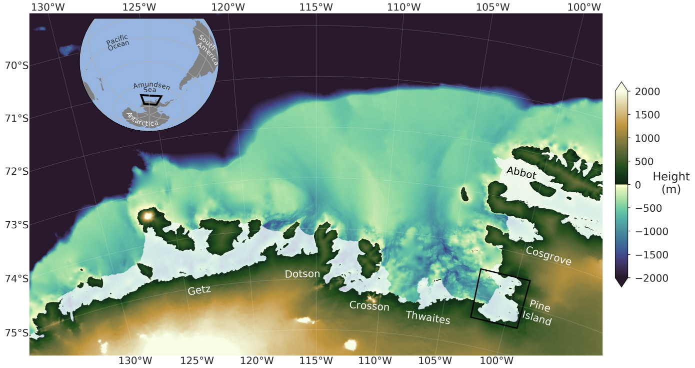
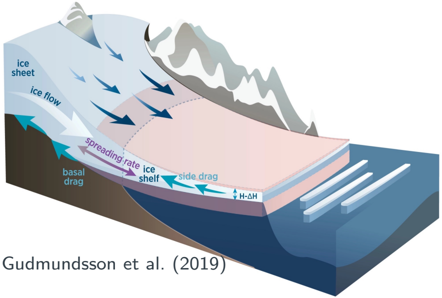


Amundsen Sea ice shelves have some of the highest meltrates in Antarctica (Adusumilli et al., 2020).



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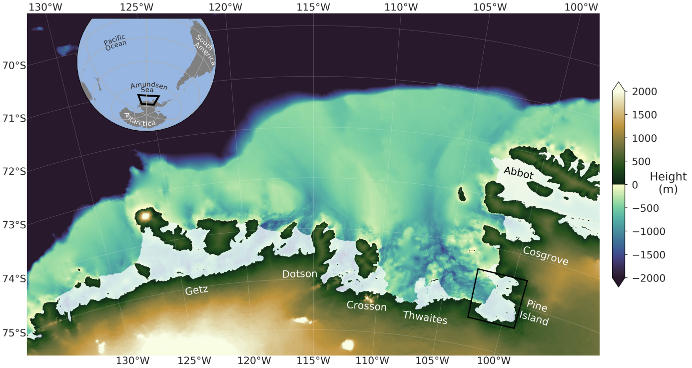

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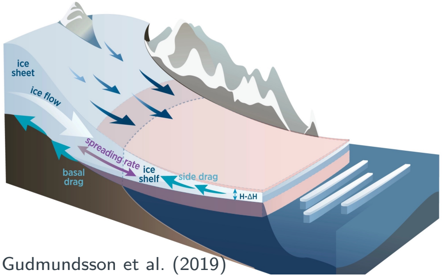

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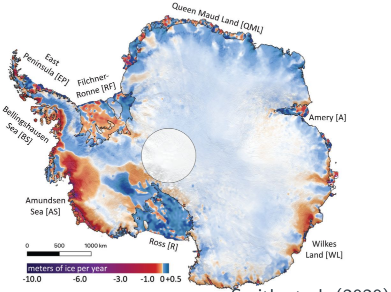

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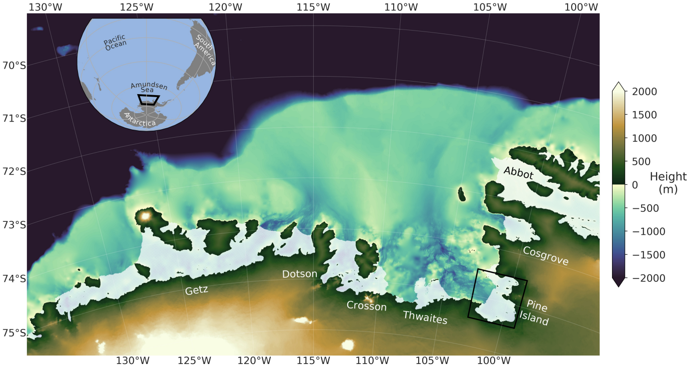

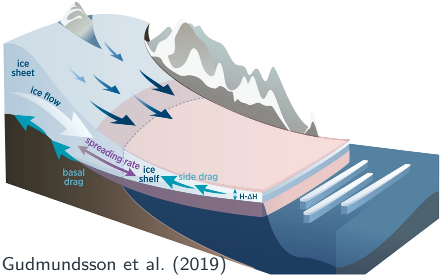

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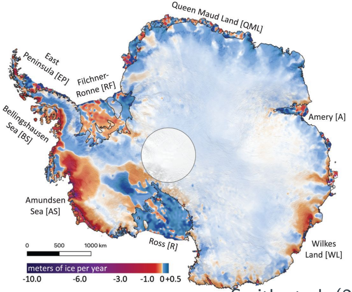

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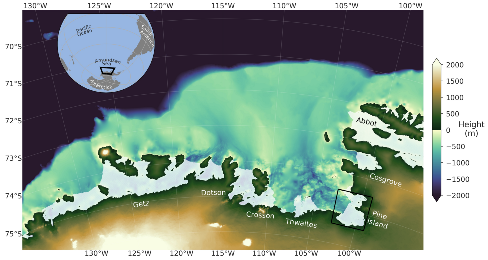
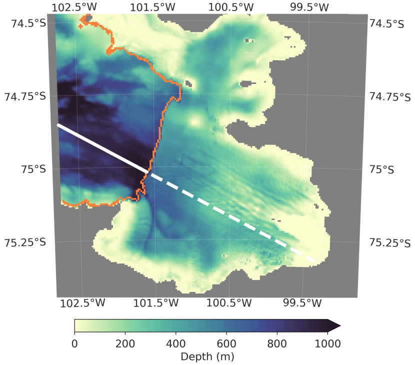
Amundsen Sea ice shelves have some of the highest meltrates in Antarctica (Adusumilli et al., 2020).

This leads to ice shelf thinning, glacial mass loss, and sea level rise (Fürst et al., 2016; Gudmundsson et al., 2019).

The Pine Island Ice Shelf





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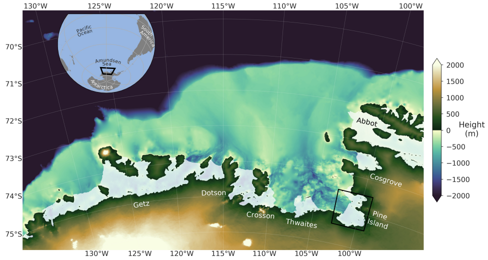

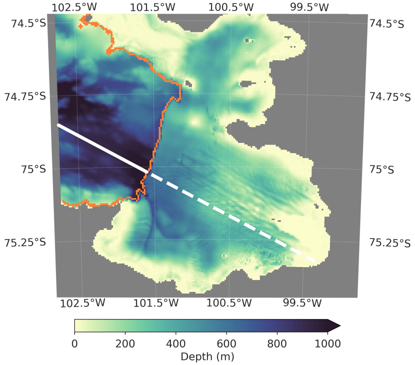

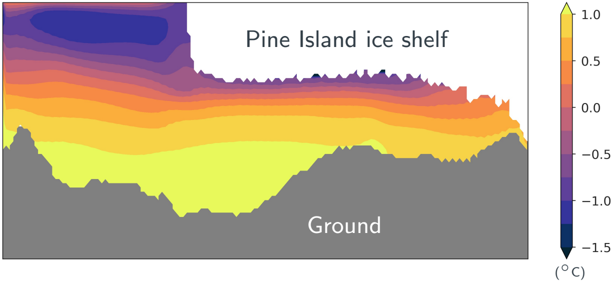

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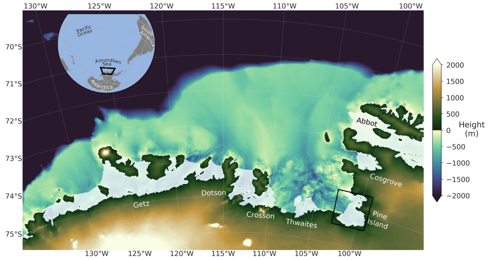

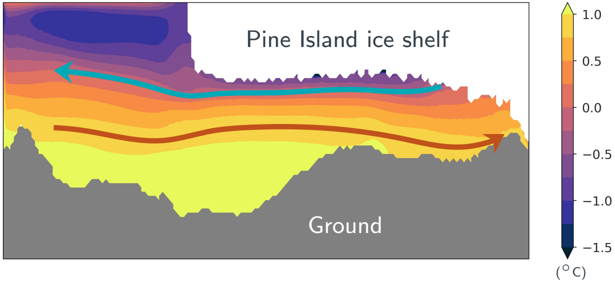

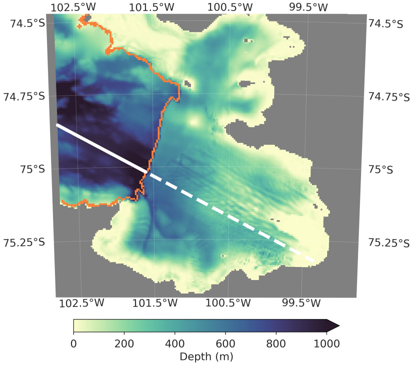

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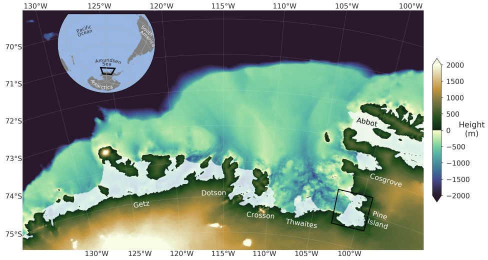
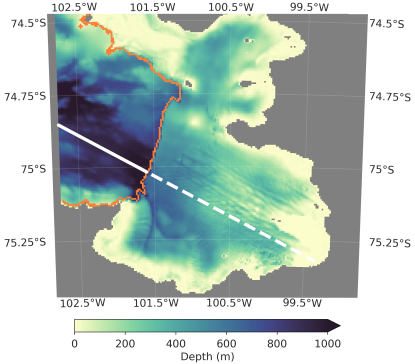
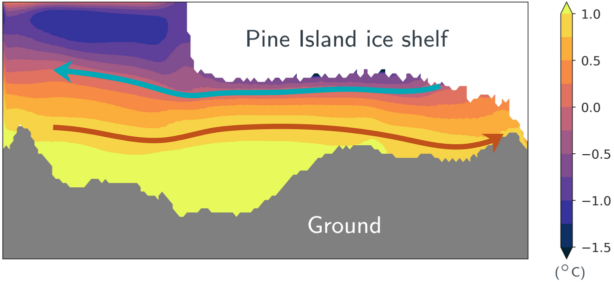
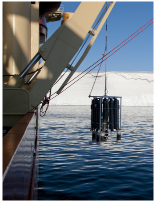


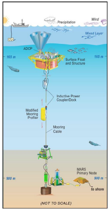


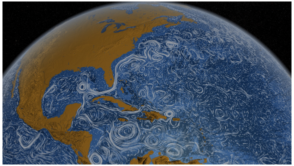


Warm waters access the deepest portions of the ice shelf and drive high meltrates

Our Contribution: Physics Informed Bayesian Inference



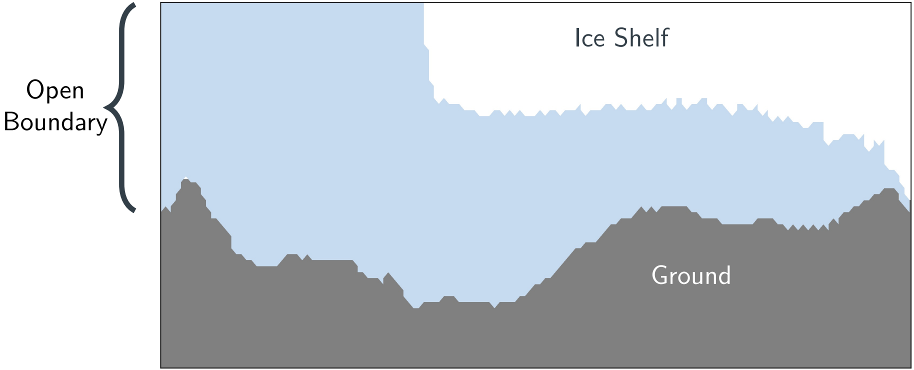




Physics Informed Bayesian Inference

How much information do we gain in our estimate of meltrates from sparse ocean observations?

A Two Stage Inference Problem for Open Boundary T, S, & U



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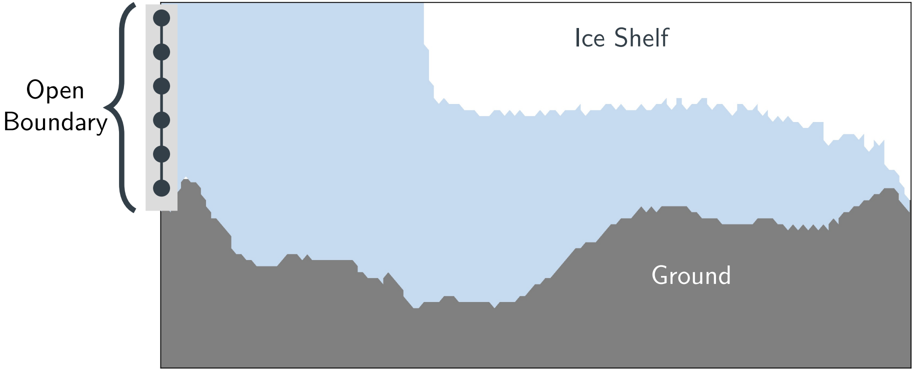

Stage 1
Optimal Interpolation

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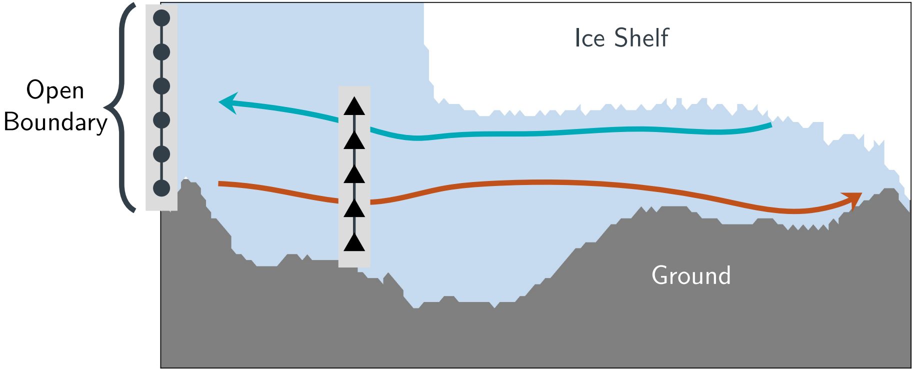
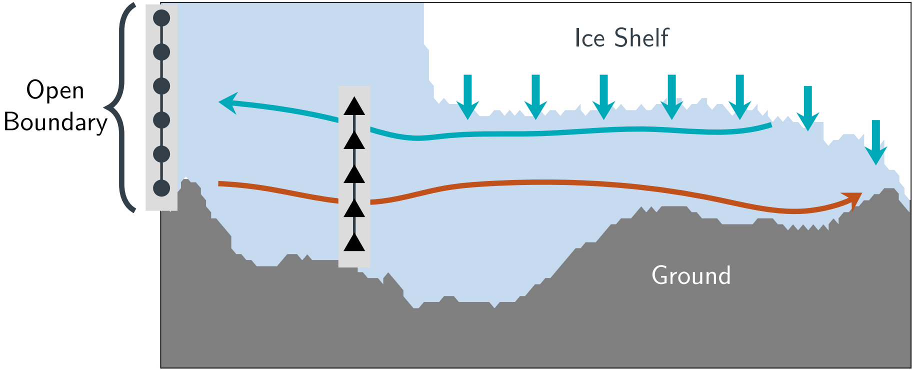


Stage 1
Optimal Interpolation

→

Stage 2
Physics Informed Inference

A Two Stage Inference Problem for Open Boundary T, S, & U



Stage 1 Optimal Interpolation → **Stage 2** Physics Informed Inference → Meltrate Uncertainty Reduction

Two Stages of Uncertainty Propagation

Optimization:

$$\begin{matrix}\text{Optimal}\\ \text{OBCS}\\ \text{Stage 1}\end{matrix} := \arg\min \left\{ \begin{matrix}\text{CTD/ADCP}\\ \text{Misfit}\end{matrix} + \begin{matrix}\text{Prior}\\ \text{Deviation \&}\\ \text{Uncertainty}\end{matrix} \right\}$$

$$\begin{matrix}\text{Optimal}\\ \text{OBCS}\\ \text{Stage 2}\end{matrix} := \arg\min \left\{ \begin{matrix}\text{Mooring}\\ \text{Misfit}\end{matrix} + \begin{matrix}\text{Stage 1}\\ \text{Deviation \&}\\ \text{Uncertainty}\end{matrix} \right\}$$

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Uncertainty Reduction / Info Gain

$$\underbrace{\Gamma_{\text{post}}}_{\substack{\text{Posterior}\\ \text{Uncertainty}}} = \underbrace{\Gamma_{\text{prior}}}_{\substack{\text{Prior}\\ \text{Uncertainty}}} - \underbrace{\sum_i \lambda_i \tilde{\mathbf{v}}_i \tilde{\mathbf{v}}_i^T}_{\substack{\text{Stage 1}\\ \text{Info Gain}}} - \underbrace{\sum_j \mu_j \tilde{\mathbf{u}}_j \tilde{\mathbf{u}}_j^T}_{\substack{\text{Stage 2}\\ \text{Info Gain}}}$$

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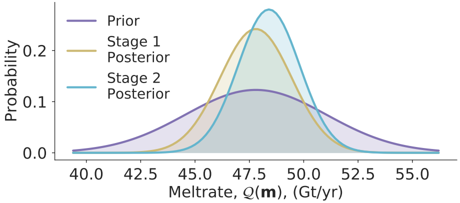

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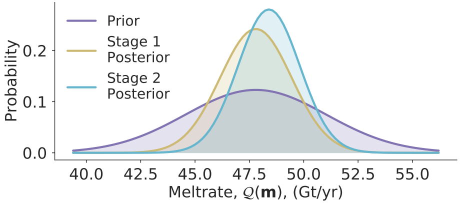

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- Optimization gives mode OBCS estimate
- Randomized EVD for scalable, “Gaussianized” estimate of uncertainty around mode
- Adjoint for efficienty uncertainty propagation

Two Stages of Uncertainty Propagation

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$$\underset{\text{Stage 1}}{\text{Optimal OBCS}} := \arg\min \left\{ \text{CTD/ADCP Misfit} + \text{Prior Deviation \& Uncertainty} \right\}$$

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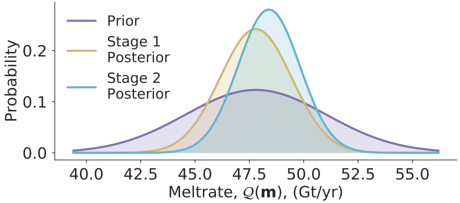

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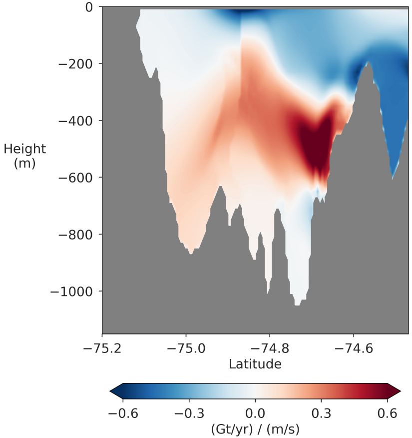
- Eigenvectors show regions informed by observations
- Sensitivity shows important regions for meltrate

Meltrate Uncertainty Reduction



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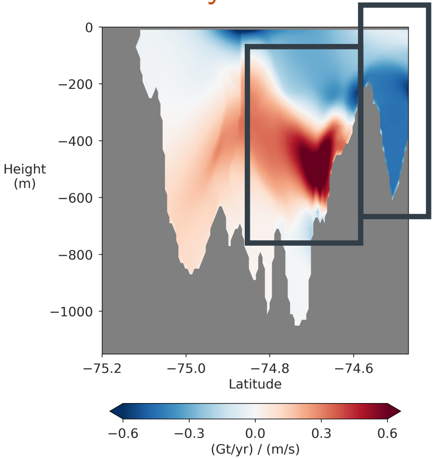

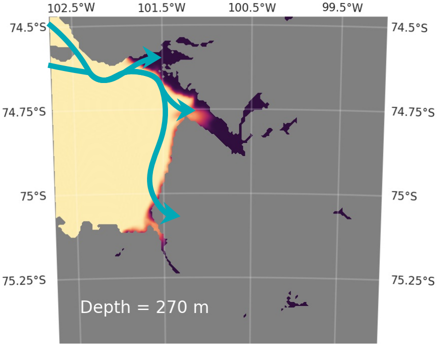

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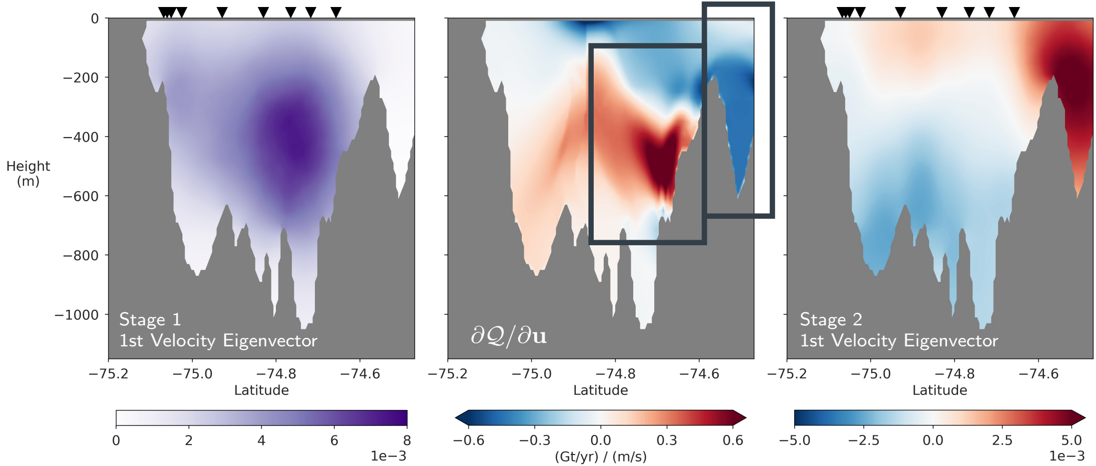

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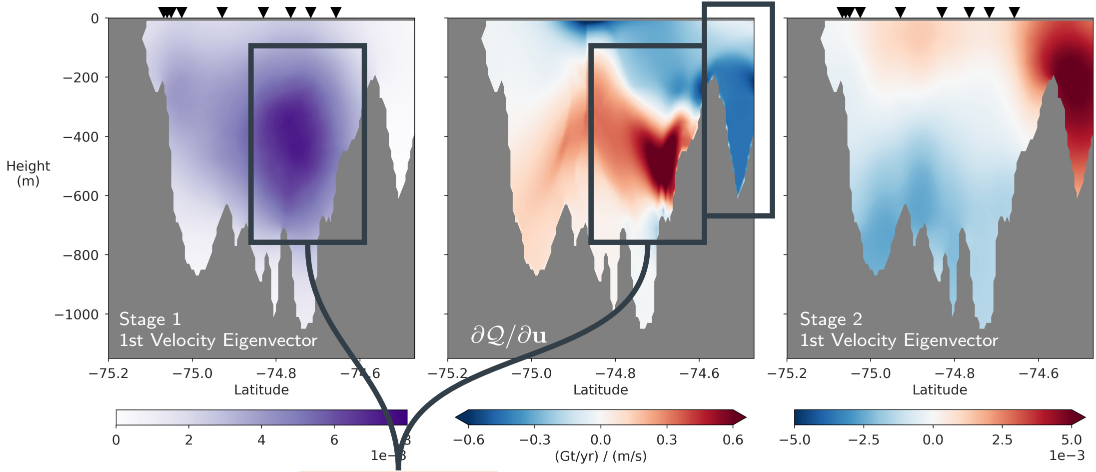
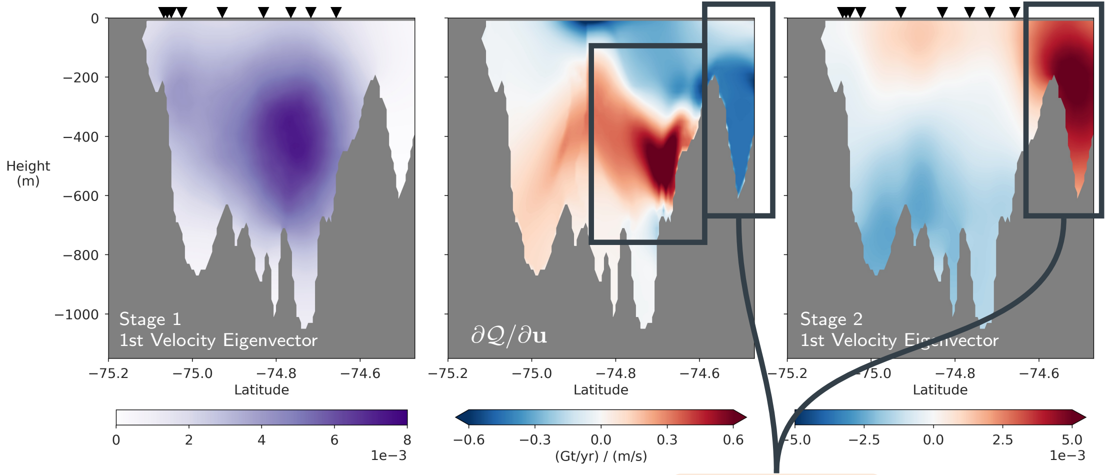


$$\frac{\lambda_1}{\lambda_1 + 1}(\tilde{\mathbf{v}}_1^T \mathbf{q})^2$$

Stage 1 Info Gain

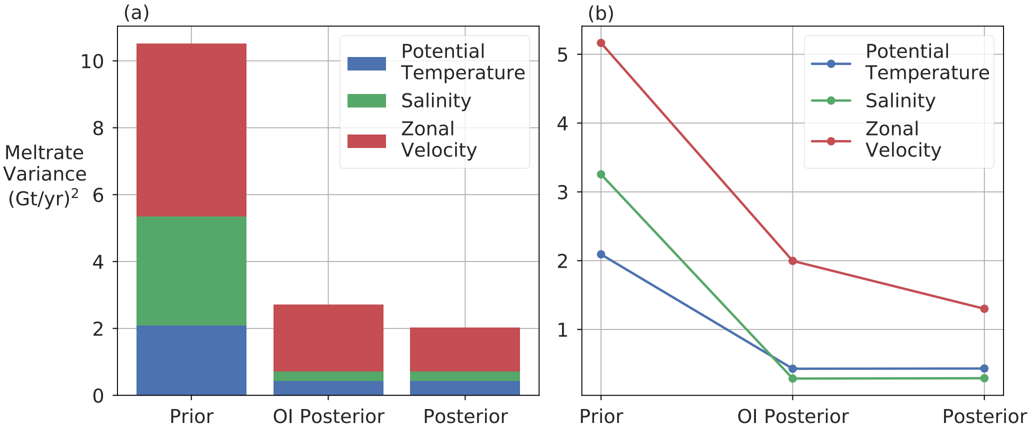
Meltrate Uncertainty Reduction



$$\frac{\mu_1}{\mu_1 + 1}(\tilde{\mathbf{u}}_1^T \mathbf{q})^2$$

Stage 2 Info Gain

Uncertainty Quantification of Ocean Driven Melting



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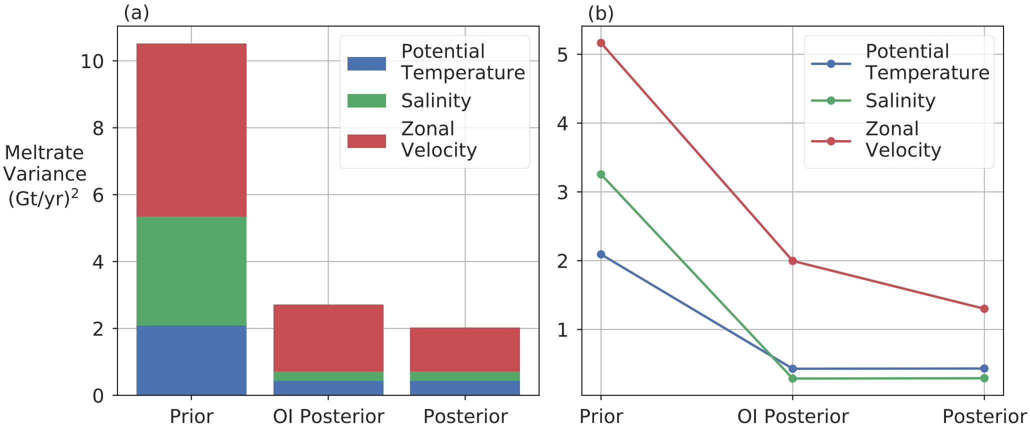
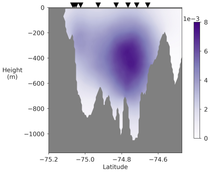


- Second stage: information gain via propagation onto unobserved variable, and to meltrate
- Standard deviation reduced by $\sim$90% relative to prior 3.2 Gt/yr

Summary & Conclusions

- CTD & LADCP data provide good constraints on conditions across most of Pine Island Bay



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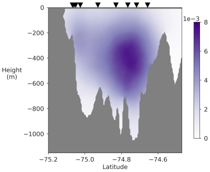

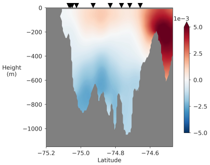

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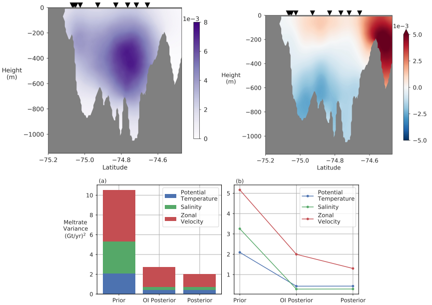

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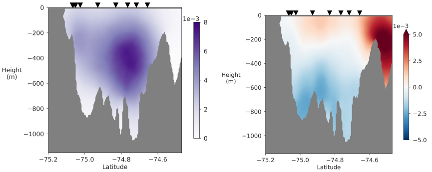
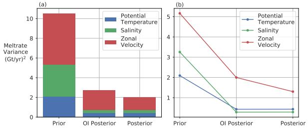
- Unobserved regions informed via model dynamics, propagating hydrographic information onto the circulation and meltrate field
- Uncertainty in meltrate reduced by 90% relative to prior standard deviation of 3.2 Gt/yr
- To stay faithful to the math, **no adjustments** made to the framework once inference has begun!





Limitations (a.k.a. Ongoing & Future Work)

- Total meltwater flux (48.4 Gt/yr) smaller than satellite observations
- Uncertainty estimate only accounts for open boundaries
- Linearity assumption behind sensitivity, uncertainty estimate is suspect
- Temporal variability not considered in modeling framework

References I

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